Evaluation of an Order Parameter for the Reaction-Diffusion Model in a Cellular Automaton

How does an order parameter perform on a Reaction-Diffusion model implemented in a Cellular Automata?

Creation of Gang Territories and other Patterns

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1. Introduction

The goal of Pattern Simulation in Computer Science is to generate different kinds of patterns from different models.

The Reaction-Diffusion model [1], in Cellular Automata [2], generates Turing patterns [3] that are very interesting in different areas of science.

To monitor the state of these patterns, this project will investigate using an order parameter [4] to tell how segregated the world is.

3. Research question

How does an order parameter perform on a Reaction-Diffusion model implemented in a Cellular Automata?

Sub-questions:
Q1. Model creation
Q2. Evaluation of order parameter
Q3. Asserting validity for bigger parameters
Q4. Limitations of Order parameter

5. Conclusion

An order parameter provides helpful information about the state of the world (segregation, convergence...)

ε(t) is a good order parameter for our Reaction-Diffusion model.

It has its properties as well as limitations, and more research can be done to further explore them.

2. Background

A model for simulating this behaviour in 2D (and 3D) cellular automata already exists, where a cell can only be 0 or 1 based on two radii ri and ro [1]:

$$s_{t+1}(C) = \begin{cases} 1 & \sum_{C_i} C_i * \omega_t(C_i) > 0 \\ 0 & \sum_{C_i} C_i * \omega_t(C_i) \leq 0 \end{cases}, \text{where}$$

$$\omega_t(C_i) = \begin{cases} 0 & (x-x_i)^2 + (y-y_i)^2 > r_o^2 \\ 1 & (x-x_i)^2 + (y-y_i)^2 \leq r_i^2 \\ -1 & \text{otherwise} \end{cases},$$

where
C is a cell of the Cellular Automata
and
C_i is a neighbor of C

Figure 1: Equation formulation for the reaction - diffusion model in a CA

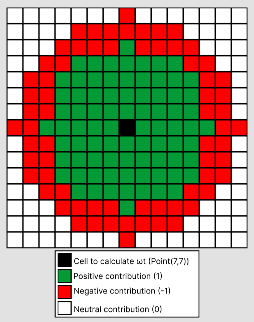

Figure 2: example of ωt(7,7) with ri = 5, ro = 7

Two initialization of the cells: random and segregated (random 20×20 patch of activated cells).
Order parameter formula, adapted from random-walker model [2]:

$$\mathcal{E}(t) = \frac{1}{n^2 \cdot |N(x,y)|} \cdot \left| \sum_{(x,y)\in CA} \sigma(x,y) \sum_{(\tilde{x},\tilde{y})\in N(x,y)} \sigma(\tilde{x},\tilde{y}) \right| \quad (3)$$

where
n is the length of the grid

$$\sigma(x,y) = \begin{cases} -1 & \text{if} \quad CA(x,y) = 0 \\ 1 & \text{if} \quad CA(x,y) = 1 \end{cases} \quad (4)$$

$$N(x,y) = \begin{cases} N_4(x,y) & = \{(x\pm1,y),(x,y\pm1)\} \\ N_{Dyn}(x,y) & = \{(\tilde{x},\tilde{y}) \mid (x-\tilde{x})^2 + (y-\tilde{y})^2 \leq r_i^2\} \end{cases} \quad (5)$$

Figure 3: Equation formulation for the order parameter

Expected behavior of ε(t):

Well-mixed state: ε(t) ≈ 0
Segregated state: ε(t) ≈ 1

Two neighborhoods N(x,y):

N4(x,y)

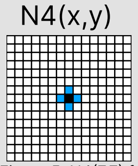
Figure 5: N4(7,7) for ri=5 and ro=7

NDyn(x,y)

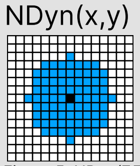
Figure 5: NDyn(7,7) for ri=5 and ro=7

4. Results

Q1. Model creation

The model creates Turing patterns for some arbitrary parameters with random start. not all ro and ri provide Turing patterns

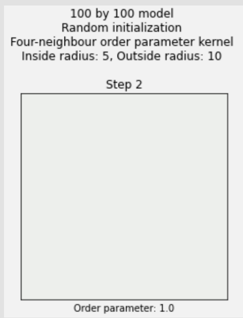

Figure 4: Example segregated state

Figure 5: Example non-segregated state

Q3. Bigger parameters

The model creates patterns when the domain and parameters size increase.

The parameter discovery method, with the equation ro ≈ √2 · ri, is a very accurate one

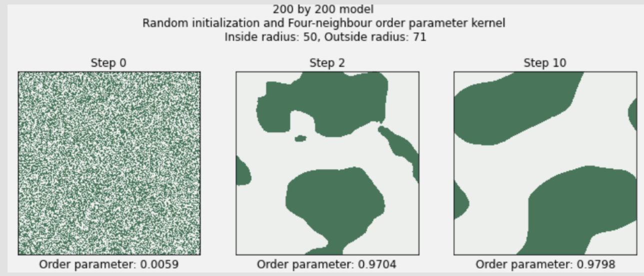

Figure 9: Simulation for bigger parameters in the model

Q2. Evaluation of Order parameter:

Phase Transition:

Non-segregated states for arbitrary values: which parameters produce patterns with ε(t)

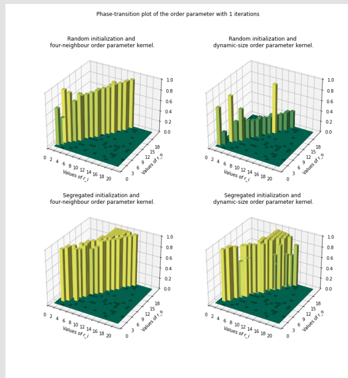
Figure 6: Phase-transition plot

Linear regression:

Parameters have linear trend:
ro ≈ √2 · ri, ε(t) monitors this

Edge cases in the segregated states.

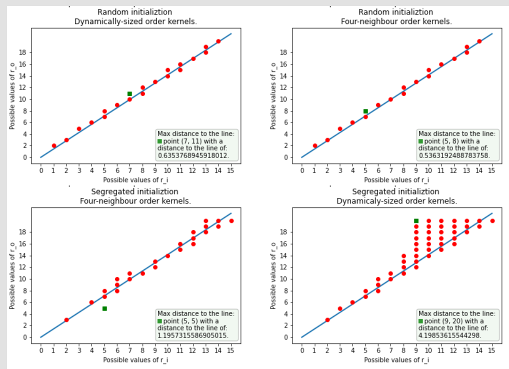

Figure 7: Linear Regression plot

Evolution:

Once interesting pairs of parameters found, study their evolution over time with the use of ε(t).

Similar behavior to the paper ε(t) is adapted from [2]

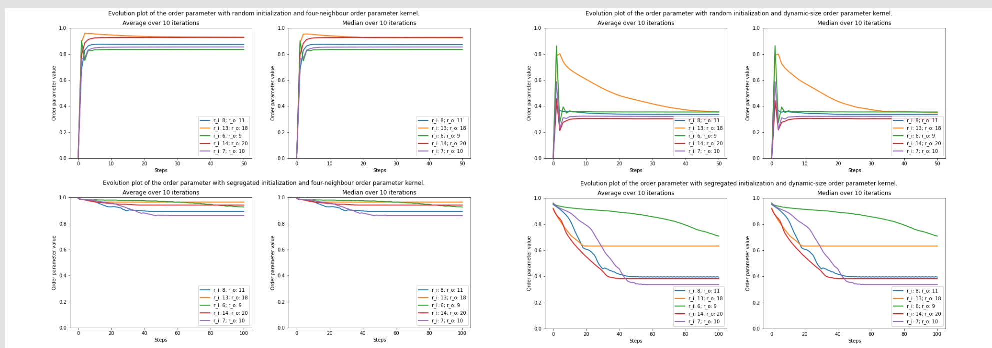
Figure 8: Evolution plots

Q4. Limitations of Order Parameter

ε(t) is useful to control segregation state of the world, but bad for other cases.

One example: Turing Shape Classification experiment

ε(t) gives bad performance for thsi experiment so we try a new parameter.

Relative Area = activated cells / total cells

Figure 10: Classification experiment

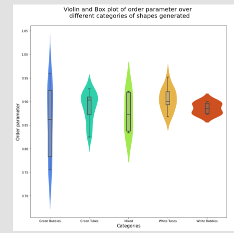
Figure 11: ε(t) violin plot

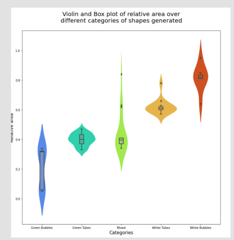
Figure 12: Relative area violin plot

References

[1] Skrodzki, Martin & Polthier, Konrad. (2017). Turing-Like Patterns Revisited: A Peek Into The Third Dimension.
[2] Downey, A. B. (2018). Cellular Automatons, Game of Life. In Think complexity: Complexity science and computational modeling (pp. 67–99). essay, O'Reilly.
[3] Turing, A. M. (1952) The chemical basis of morphogenesis, Biological Sciences 237, 641 (1952), 37–72. URL: http://www.jstor.org/stable/92463
[4] Alsenafi, A., & Barbaro, A. B. T. (2018). A convection–diffusion model for gang territoriality. Physica D: Nonlinear Phenomena, 510, 765–786. https://doi.org/10.1016/j.physa.2018.07.004