Physics-Informed Neural Networks with Adaptive Sampling for Option Pricing

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01 Background

PINNs

- Physics-informed neural networks
- Solve partial differential equations

RAD sampling

- Residual-based adaptive distribution sampling
- Focus on training areas where the loss is greatest

Black-Scholes

- Models the price of stock options

02 Objective

How does Residual-based Adaptive Distribution Sampling affects the performance of Physics-Informed Neural Networks in solving Black-Scholes?

- How do other methods for solving Black-Scholes perform?
- Which type of option befits most from RAD sampling?

03 Methodology

PINN loss function

$$\mathcal{L} = w_{pde}\mathcal{L}_{pde} + w_{init}\mathcal{L}_{init} + w_{b0}\mathcal{L}_{b0} + w_{b1}\mathcal{L}_{b1}$$

RAD PDF

$$p(x) \propto \frac{\varepsilon^k(x)}{\mathbb{E}[\varepsilon^k(x)]} + c$$

Black-Scholes PDE

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV$$

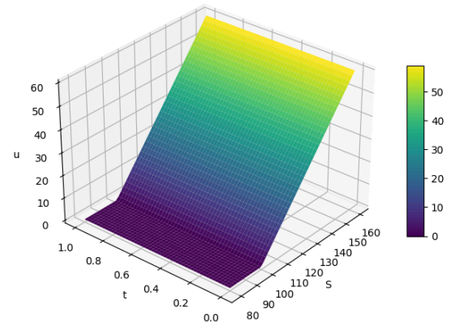
Boundary Conditions

$$C(0,t) = 0$$
$$C(S,t) = S - K \quad \text{when} \quad S \to \infty$$
$$C(S,T) = \max(S-K, 0)$$

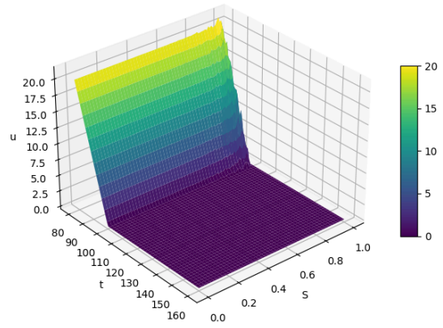
[1] Wu et al. A comprehensive study of non-adaptive and residual-based adaptive sampling for physics-informed neural networks (2023)
[2] Gatta et al. Meshless methods for American option pricing through Physics-Informed Neural Networks (2023)

04 Alternative Solutions

Simulated option prices (u) over time (t) and stock price (S)



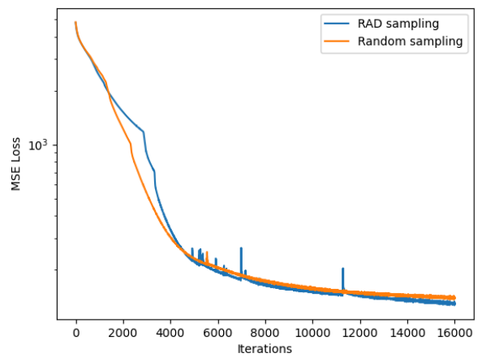
European Call Option
(Analytical Solution)



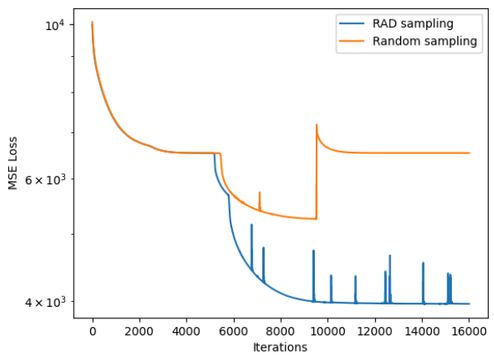
American Put Option
(Monte-Carlo Simulation)

05 Results

The loss of RAD vs non-adaptive random sampling with an average of 10 runs for each method



European Call Option



American Put Option

06 Discussion

- RAD sampling results in a consistent small improvement pricing European call options
- RAD sampling offers a large, but inconsistent improvement solving American put options

- Future work could include:
  - Multi-asset options
  - Inverse model

07 Conclusion

RAD sampling positively affects the performance of PINNs for option pricing

- European call options: 7.57% loss reduction
- American put options: 39.33% loss reduction