Evaluating Proactive, Reactive, and Hybrid Strategies for the Stochastic Multi-Mode RCPSP with Hard Deadlines

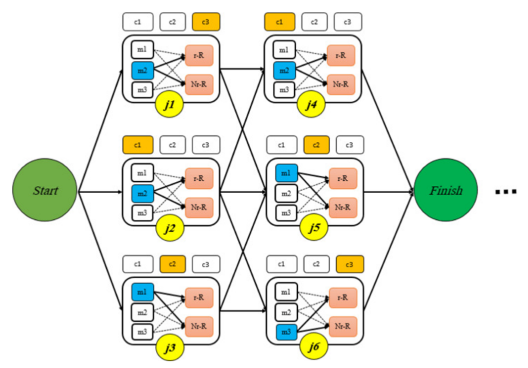
Andreas Shiamishis

Department of Electrical Engineering, Mathematics and Computer Science Delft University of Technology

Introduction

Stochastic scheduling is a crucial and rapidly growing field that attracts significant interest across numerous domains, particularly in the development of digital factories.

We focus on the stochastic Multi-Mode Resource Constrained Project Scheduling Problem (MMRCPSP) with hard deadlines. It involves assigning start times and execution modes to tasks under resource limits and precedence and deadline constraints. Each task can run in multiple modes, each with different durations and resource requirements.



Multi-mode resource constrained project scheduling problem illustration [4].

Stochastic Scheduling Algorithms

Algorithms to solve this problem lie in a spectrum that ranges from fully proactive to fully reactive approaches. The focus of this research is a proactive, a reactive and a hybrid approach [2].

- **Proactive approach:** Computes a fixed schedule in advance using sampled durations and strictly follows it during execution.
- **Reactive approach:** Starts with an initial schedule based on estimated durations and dynamically adjusts it in real-time as actual durations are revealed during execution.
- **STNU-based approach:** A hybrid method that creates a partial order schedule with built-in temporal flexibility, enabling real-time scheduling decisions in response to uncertainty.

Research Question

How does a Simple Temporal Network with Uncertainty (STNU) based approach fare against other state of the art algorithms such as proactive and reactive approaches?

- How do sampling strategies affect model performance?
- How does the variance of the durations' distributions affect performance?
- How do the approaches perform with different duration distributions (e.g., uniform vs. binomial)?
- How do the algorithms scale with larger instances (e.g., more tasks)?

Methodology

Sampling:

- Robust approach: use the upper bounds of the duration distributions
- Mean approach: use the mean of the duration distributions
- Quantile-based approach: use some quantile of the distributions as the sample, e.g. 0.9 quantile

Modelling the deadlines:

- Deadlines can be modelled as dummy tasks starting at 0 and having a duration equal to the deadline
- Tasks have an ordering constraint which states that they need to be completed before their respective deadline task is completed

Mode selection:

- Modes are chosen *offline* before scheduling.
- This simplifies computation but limits adaptability during execution.

Evaluation

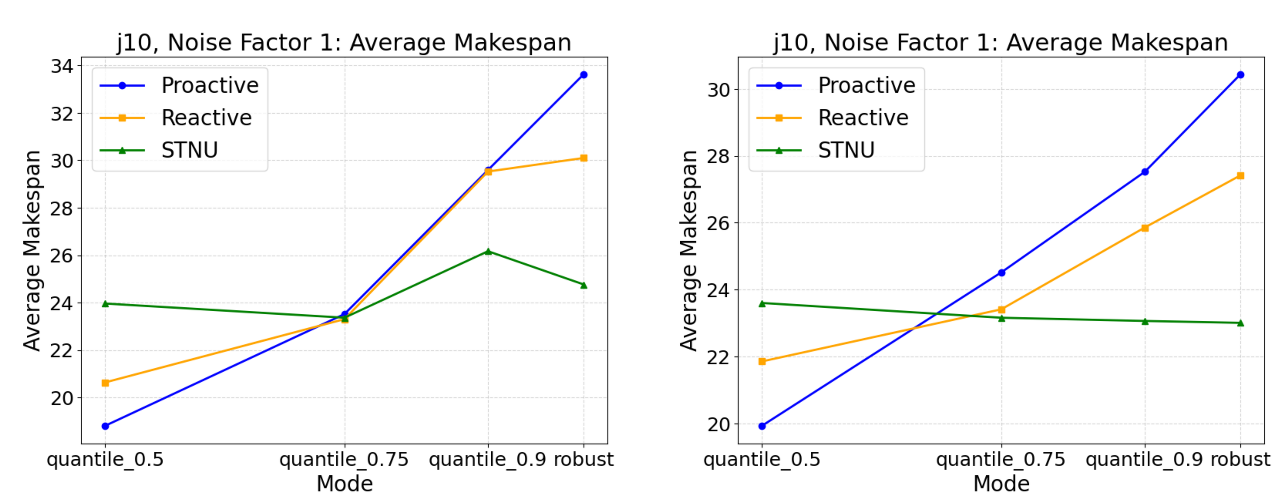
The following metrics are used:

- **Feasibility ratio:** Fraction of simulations where all constraints are met.
- **Execution time:** Includes both offline planning and online adaptation.
- **Makespan:** Total time to complete all tasks.

Experimental Process

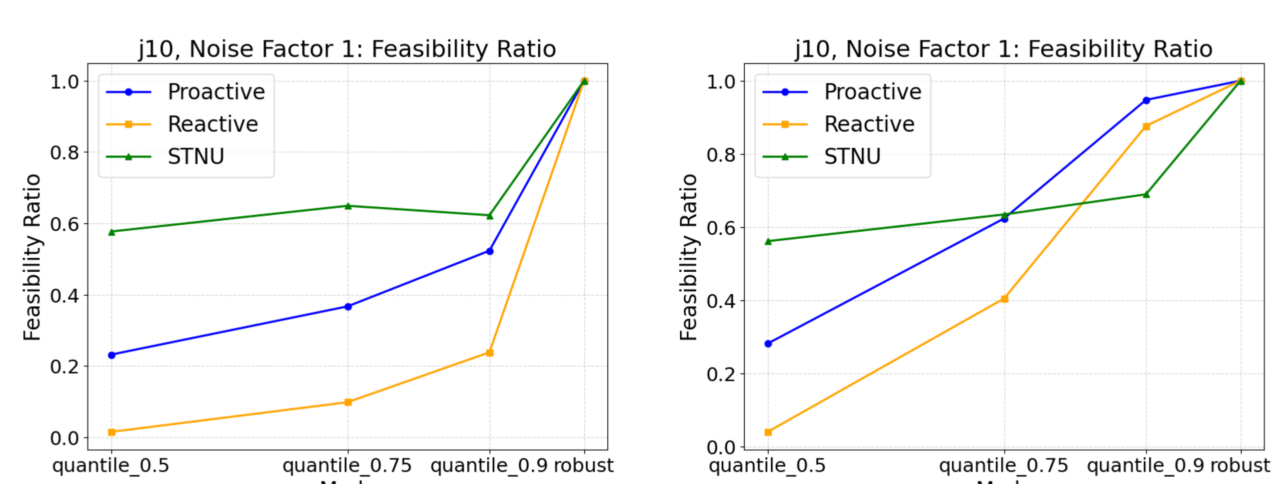
To evaluate and compare the approaches, multiple runs are conducted on instances from the PSPLib [3] with varying task counts (10 and 20), different duration distributions (uniform and binomial), and varying levels of variance. Some results are compared using the Wilcoxon Matched-Pairs Rank-Sum Test [1] for more insight.

Results



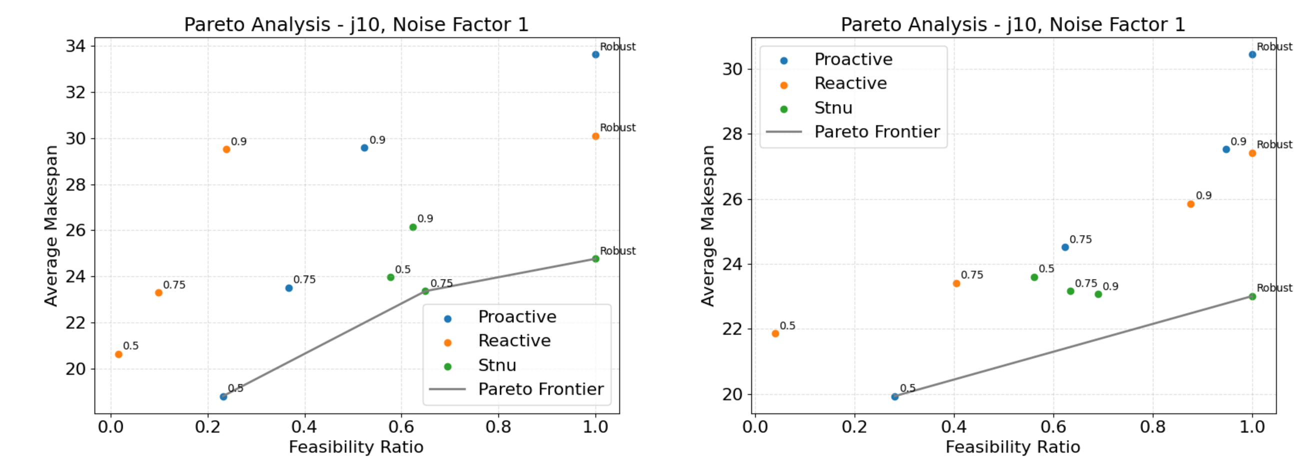
(a) Uniform distribution (b) Binomial distribution

Figure 1. Makespan values across three approaches. Results are based on 100 instances with 10 tasks and a noise factor of 1.



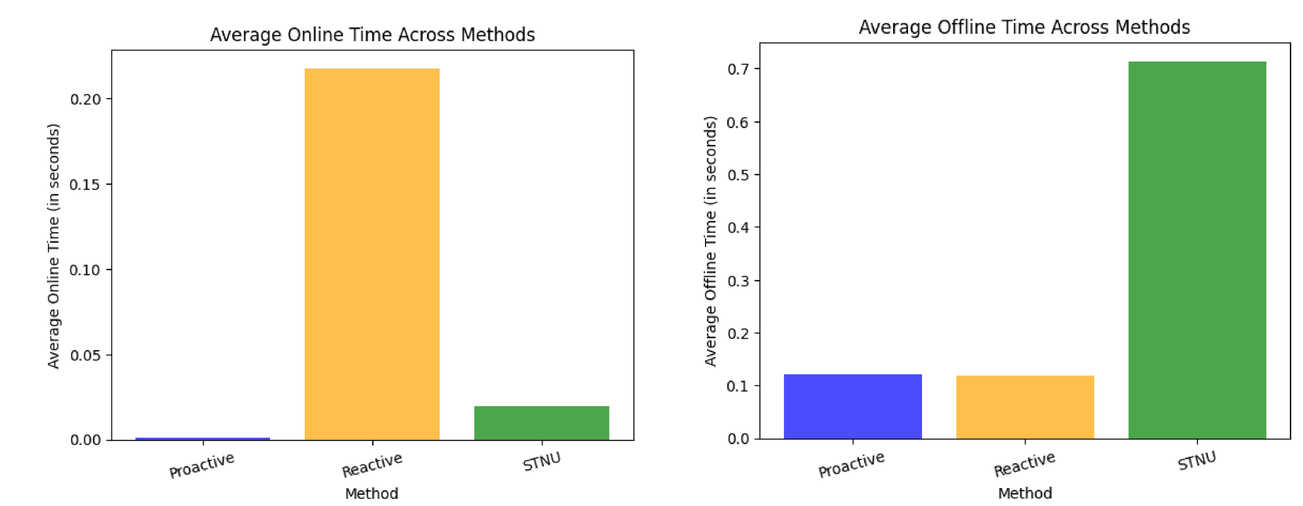
(a) Uniform distribution (b) Binomial distribution

Figure 2. Feasibility ratios across three approaches. Results are based on 100 instances with 10 tasks and a noise factor of 1.



(a) Uniform distribution (b) Binomial distribution

Figure 3. Scatter plots of feasibility ratio against makespan for all sampling methods and both uniform and binomial distributions respectively. The gray line indicates the pareto optimal frontier.



(a) Online execution time (b) Offline execution time

Figure 4. Average execution times across three approaches. Based on 100 instances, each with 10 scenarios, under two noise factors (1 and 2) and two instance sizes (10 and 20).

Methods\Quantile	0.5	0.75	0.9	1
Reactive-Proactive	Proactive	Reactive	Reactive	Reactive
Reactive-STNU	STNU	STNU	STNU	STNU
Proactive-STNU	Proactive	STNU	STNU	STNU

Table 1. Results of the Wilcoxon Matched-Pairs Rank-Sum Test between methods across different sampling quantiles for the binomial distribution. Each value considers all noise factor-instance size combinations. Cells in red indicate comparisons with non-significant differences (p-value > 0.05), using the Wilcoxon signed-rank test with $\alpha = 0.05$.

Methods\Quantile	0.5	0.75	0.9	1
Reactive-Proactive	0.12	2.46e-47	3.88e-211	1.40e-267
Reactive-STNU	0.72	0.87	5.25e-13	3.57e-150
Proactive-STNU	3.28e-12	0.67	8.51e-35	1.12e-224

Table 2. p-values from Wilcoxon signed-rank tests comparing method pairs across different quantile-based sampling strategies. The significance level is set to $\alpha = 0.05$; values in red indicate non-significant results ($p > 0.05$).

Analysis

- STNU produces better schedules for robust sampling than the other approaches.
- STNU consistently achieved higher feasibility than the other methods.
- Proactive-0.5 yielded best makespans but lower feasibility; robust sampling improved feasibility with longer schedules.
- STNU-robust and proactive-0.5 appear consistently on the Pareto frontier showcasing the tradeoff between efficiency and robustness.
- Reactive algorithm requires substantially more online time than the other two.
- STNU needs significantly more offline time, as expected.

Conclusions

- Evaluated proactive, reactive, and hybrid STNU-based methods for stochastic MMRCPSP with deadlines.
- Extended prior models to handle multi-mode tasks and strict deadlines using dummy-task enforcement.
- STNU showed strong feasibility and robustness, balancing flexibility and efficiency.
- Sampling strategy proved to be a deciding factor for model performance.
- A core conflict exists between achieving optimality and ensuring robustness.

References

[1] Edward E. Cureton. "The normal approximation to the signed-rank sampling distribution when zero differences are present". In: *Journal of the American Statistical Association* 62.319 (1967), pp. 1068–1069.

[2] Kim van den Houten et al. "Proactive and Reactive Constraint Programming for Stochastic Project Scheduling with Maximal Time-Lags". In: *arXiv preprint arXiv:2409.09107* (2024). URL: `https://arxiv.org/abs/2409.09107`.

[3] Rainer Kolisch, Andreas Sprecher, and Andreas Drexl. *PSPLIB - Project Scheduling Problem Library*. 1997. URL: `http://www.om-db.wi.tum.de/psplib/`.

[4] Reza Nemati-Lafmejani, Hamed Davari-Ardakani, and Hamid Najafzad. "Multi-mode resource constrained project scheduling and contractor selection: Mathematical formulation and metaheuristic algorithms". In: *Applied Soft Computing* 81 (2019), p. 105533. ISSN: 1568-4946. DOI: `https://doi.org/10.1016/j.asoc.2019.105533`. URL: `https://www.sciencedirect.com/science/article/pii/S1568494619303035`.