Territory patterns created by random walker agents in three-dimensions

A. de Bruijn | Author
CSE3000 | Course
28 June 2023 | Date
M. Skrodzki, A.B.T. Barbaro | Supervisors
A.debruijn-3@student.tudelft.nl | Contact

1. Introduction

Two-dimensional (2D) territory formation have been studied in depth in for micro-organisms

The purpose of this paper is to show how a random walker model behaves in three dimensions with different strengths of avoidance for opposing species.

Main research question:

"What similarities exist between properties of the two-dimensional and three-dimensional lattice models?"

2. Background

K. Pearson popularised the random walk in 1905. Still a hard problem by amount of data generated [1].

In 1921 **G. Pólya** proved that higher dimensional random walkers are less stable [2].

A dual-layered walker model for human gang territories was proposed by **A. Alsenafi** and **A.B. Barbaro** (hereafter refered to as the 2D territory paper). One layer is dedicated to agents walking on a cyclic 2D lattice. The other layer serves to store markers that indicate how popular a given node is for each species [3].

3. Method

This paper proved that the two dimensional territory paper can be extended to any number of dimensions. The formulae from the previous territory paper are used with references to a 3D lattice.

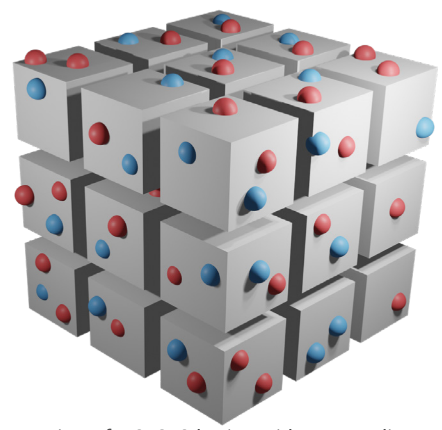

Fig 1. Representation of a 3x3x3 lattice with agents dispersed

Visualisation

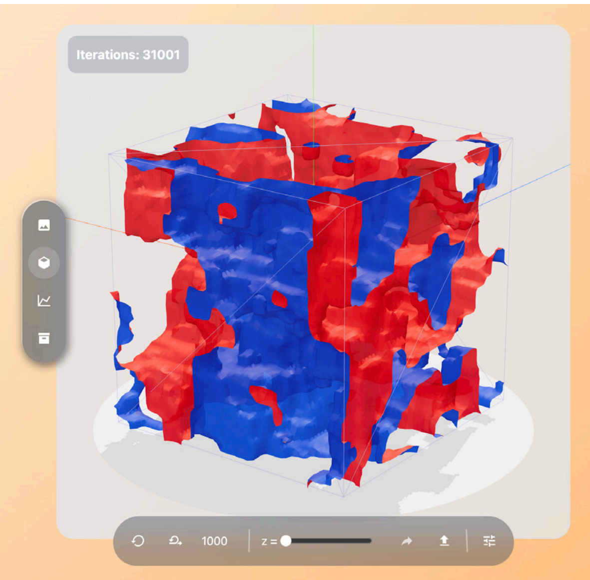

Fig 2. Visualisation with marching cubes to show the borders

The visualisation displays territory borders for dominant species using the marching cubes algorithm [4].

4. Results & Analysis

Phase transition with beta:

Critical beta (β*) is the point at which the final order parameter is 0.01. Analysis in the paper describes that the final order parameter can be well approximated when taking 50.000 iterations.

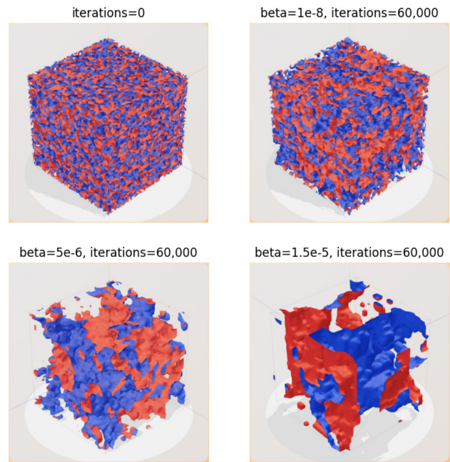

Fig 3. State for different beta values

The 2D territory paper identified that more mass results in critical beta reached earlier. Observe how this is present for a mass of 1.6M to 3.2M

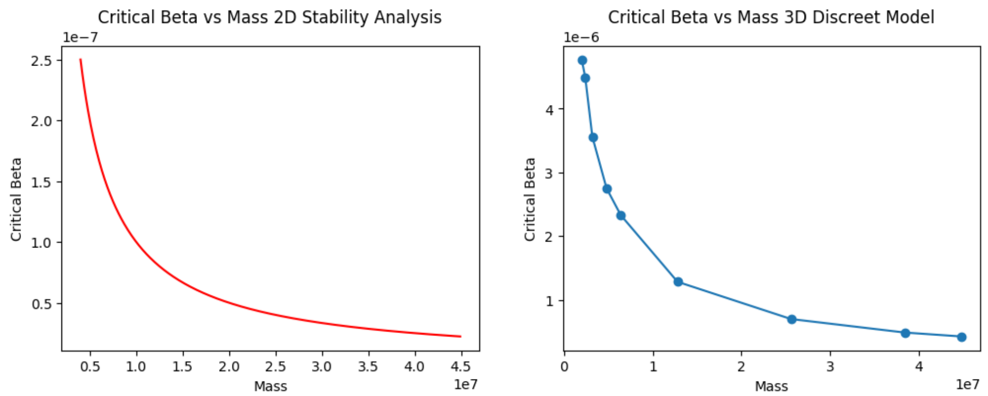

Fig 4. Critical β for different total masses.

The 2D territory paper identified that higher gamma/lambda ratio critical beta threshold reached earlier. The ratio increase also speeds up the order process.

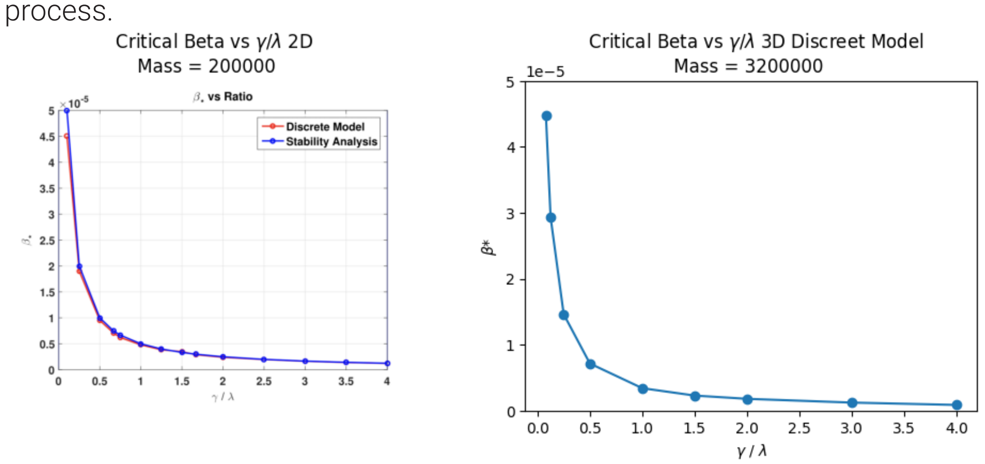

Fig 5. Critical β for different ratios of γ / λ

5. Discussion

A noteworthy difference is that the 3D model needs significantly more beta to reach the same critical point for each total mass. The same holds for comparing the behaviour of γ / λ ratio. The main cause for this large increase must be linked to the number of dimensions an agent can walk towards. Other smaller factors may also be involved. Discrete models are useful for exploring new behaviours. Continuous models provide better representations for comparing models [5].

6. Conclusion

We conclude that the main question can be answered that both models have a well- mixed and segregated state. The point at which the state changes from well-mixed to well-segregated is in both models determined by the amount of mass present in the lattices and the ratio between γ / λ plays a large role. For both models, the more mass or γ / λ they have, the less beta is required to reach the critical point. Additionally, the 3D model needs significantly more beta to reach the same critical point for the same amount of mass or γ / λ ratio.

References

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