# Solving the Multi-Level Bin Packing problem with Time Windows using Integer Programming

**Which Integer Programming models perform best when finding an optimal solution to the problem?**

CSE3000 Research Project by Max Le Blansch - Supervision by M.G. Horn & N. Yorke-Smith - 20/06/22

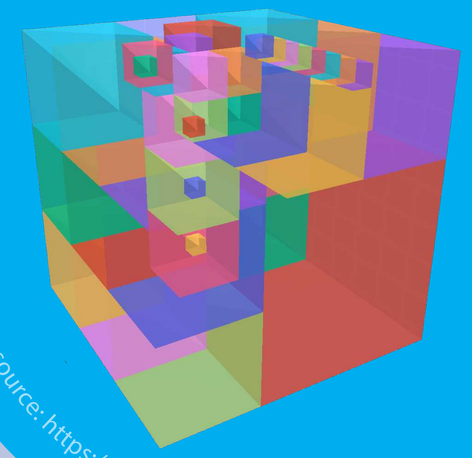
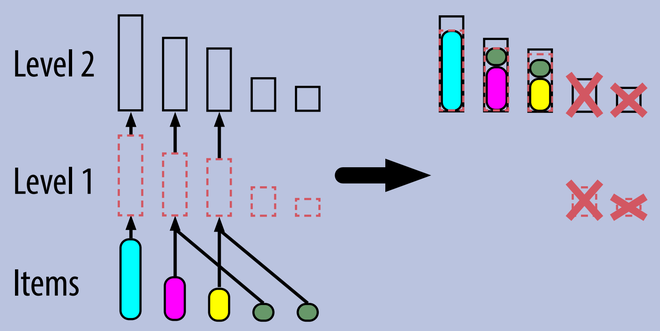


Image source: https://3dbin.alpacku.com/

## 1 Background & Method
### Problem description & Tools

**The Multi-Level Bin Packing Problem (MLBP)**

- $n$ items with a certain size
- $m$ levels
- Bins at each level with certain capacity and size
- Assign item and bins to next level bins without exceeding capacity
- Minimize sum of cost of used bins




**The MLBP problem with Time Windows (MLBPTW)**

- Add time window to items
- Items in same bin need common packing moment
- Minimize cost of used bins + lateness penalty of packing time

**Integer Programming (IP)**

- Used to solve NP-hard problems like MLBP(TW)
- Find solutions while minimizing cost function
- Model problem in terms of linear (in)equalities with integer decision variables only
- Branch-&-Bound: Divide in subproblems and solve recursively

**CPLEX**

- Tool to solve IP models
- Framework in C++
- Uses internal performance optimization methods

## 2 Models
### Definition for IP

**Compact model for MLBP**

Variables:
- Binary $x$ for direct assignments between levels
- Binary $y$ for marking the used bins

Constraints:
- Force items into a bin for $x$
- Only assign bins with $x$ that are used $y$
- Used bins $y$ can be assigned to 1 bin

**Compact model for MLBPTW**

- Extends compact model for MLBP

Variables:
- Binary $z$ for indirect assignment of items
- Integer $u$ for earliest packing time of items

Constraints:
- Force item into a bin at each level for $z$
- Transitivity of bin assignment for $z$
- No items with no time overlap packed together for $z$
- Packed together items have $u$ in latest interval

**Network flow model for MLBP and MLBPTW**

- Extends compact model for MLBP and MLBPTW similarly

Variables:
- Integer $f$ for flow amount between assignments

Constraints:
- Only flow $f$ between assingments $x$
- Every item provides 1 flow unit $f$ to the network
- Top level bins combined consume $n$ flow units $f$
- Bins not in top level have same in- as outflow units $f$

## 3 Results
### Experiments

Figure 1:

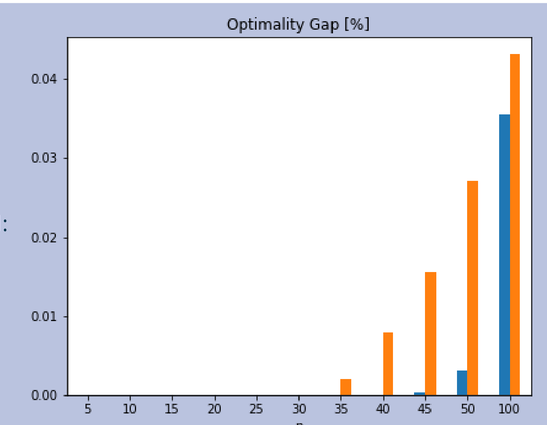
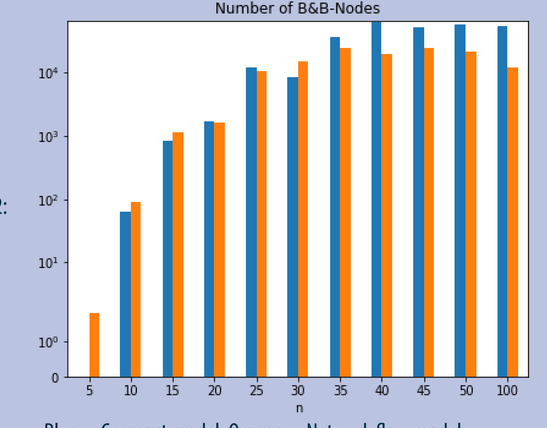



Figure 2:




Blue = Compact model, Orange = Network flow model

**Conclusion**

- Compact model for MLBP better in terms of optimality gap
- Network flow model better in terms of number of Branch-&-Bound nodes for MLBP & MLBPTW

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**Poster Template: Sterre Lutz**