Domain-specific heuristic augmentation of SAT solvers on prize-collecting job scheduling problem

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1. Introduction

- Prize Collecting Job Scheduling Problem with One Common and Multiple Secondary Resources

In PC-JSOCMSR, a collection of jobs is provided and each job is associated with a prize. A subset of the maximum total prize is chosen and scheduled among these jobs. Each job has its own set of time windows, and it can only be processed in the range of one of them. Each job requires two resources for processing: a common resource that all jobs require for a portion of their processing, and a secondary resource that is shared by just a subset of the other tasks but is required for the entire processing period. At any time instance, each resource can only process one job.

Instance:

j	p_j	p_j^{pre}	p_j^0	q_j	s_j	W'_j
1	4	1	2	1	2	{[0, 8]}
2	4	1	2	1	4	{[0, 8]}
3	4	0	3	2	3	{[3, 8]}
4	5	1	3	2	2	{[8, 14]}

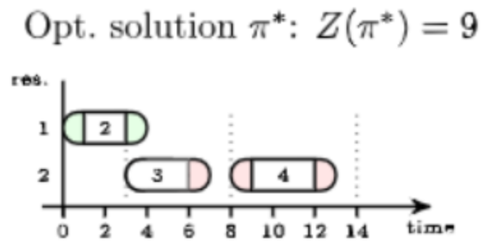


Fig 1. Example of a PC-JSOCMSR instance

2. Applications

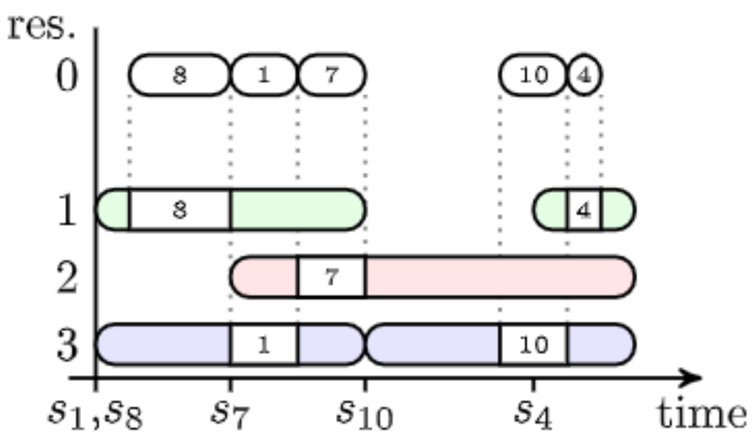
The two main applications of PC-JSOCMSR are in the avionics and particle therapy fields.

Particle Therapy Scheduling

Avionics Scheduling

3. Research Question

The main research question that we are working on in this article is to check whether it is possible to improve the performance of a SAT solver approach to PC-JSOCMSR, by heuristically augmenting it with a previously calculated lower bound solution.



A feasible schedule of jobs 1; 4; 7; 8; 10 of a PC-JSOCMSR instance with n¼10 jobs and m = 3 secondary resources

6. Conclusion

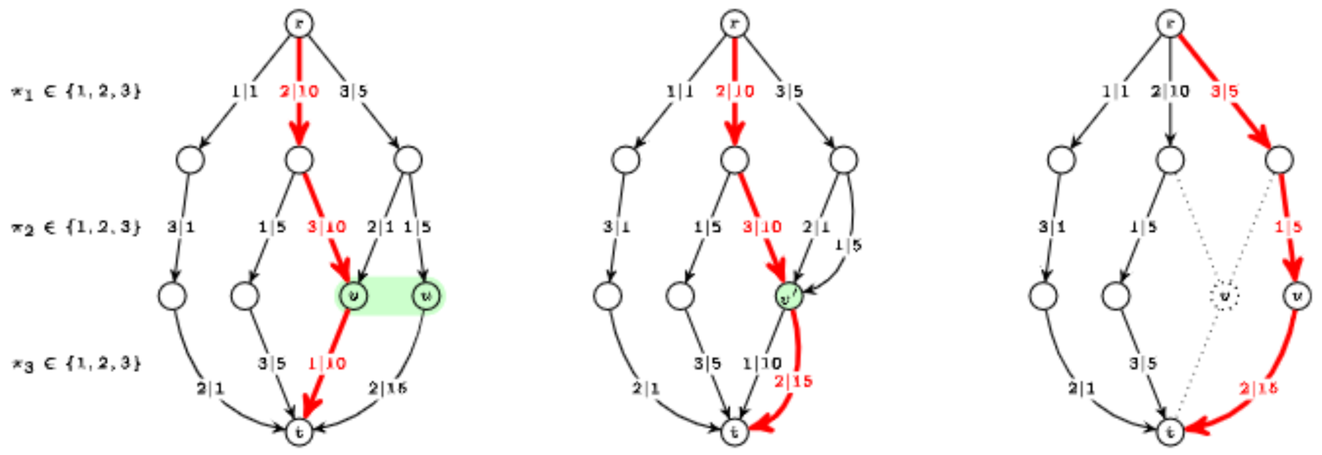
After we reviewed all the results, we observe a significant improvement in calculating the lower bound of the prize. Namely, when a WCNF encoding is passed to the solver together with a list of hints, obtained by the heuristic approach in a low amount of time, the solver manages to find a schedule with a higher prize than the cases in which no hints are provided.

References

[1]Matthias Horn, Günther Raidl, and Christian Blum. Job sequencing with one common and multiple secondary resources: An a∗/beam search based anytime algorithm. *Artificial Intelligence*, 277:103173, 09 2019.

4. Methodology

We start with implementing a domain-specific heuristic for the problem, introduced by Horn et al.[1]. Then, we encode the problem to CNF and compile the obtained formula with a SAT solver. The final and most important step of the research is to optimize the SAT solver by applying the lower bound obtained from the heuristic approach, with an objective to decrease the number of variable selections done by the solver.



(a) Exact MDD (b) Relaxed MDD (c) Restricted MDD

Fig 2. Examples of an exact, a relaxed, and a restricted MDD for a sequencing problem with ground set 1, 2, 3

$$\forall j \in J, \forall t \in T_j, s_{jt} = \begin{cases} 1 \text{ If the job } j \text{ starts at } t \\ 0 \text{ If the job } j \text{ does not start at } t \end{cases}$$

a) Starting Time Clauses

$$\forall j \in J, \forall t \in T_j, s_{jt} \implies sr_{jrt'}$$

a) Secondary Resource Clauses

$$\forall j \in J, \forall t \in T_j, s_{jt} \implies cr_{jt'}$$

b) Common Resource Clauses

$$\forall j \in J, \sum_{t=T_{j0}}^{T_j} s_{jt} <= 1$$

a) Starting Time

$$\forall j \in J, \forall r \in R, \sum_{t=T_{j0}}^{T_j} sr_{jrt} <= 1$$

b) Secondary Resource

$$\forall j \in J, \sum_{t=T_{j0}}^{T_j} cr_{jt} <= 1$$

c) Common Resource

At-Most-One Cardinality Constraints

As a final step, we heuristically augment the solver by modifying the initial lower bound model based on the LB solution from the heuristic.

5. Results

	Heuristic	*SAT*	*Aug. SAT*	*Equal*
Heuristic vs SAT	315	**324**	-	104
Heuristic vs Aug. SAT	221	-	**468**	113
SAT vs Aug. SAT	-	188	**336**	199

Fig 3. Pairwise comparison of approaches. A count of prize value closer to optimum is reported per comparison