Solving Integer Programming Models for the Multi-Level Bin Packing Problem with Conflict Constraints (MLBPCC)

1 Research Question

Do flow based IP models outperform other kind of IP models to solve MLBP(CC) instances in terms of solution time and the number of branch-and-bound nodes required?

2 Background

- Integer Programming (IP)
  - Mathematical model where the decision variables are integers
  - Decision variables are restricted by (in)equality constraints
  - Objective is to minimize/maximize a linear objective function
- MLBP
  - n items & m levels -> put all items into bins of every level
  - Minimize total bin cost
  - Tons of real life applications: packing of items to boxes which then need to be packed to containers
- Ticks – deterministic unit of work
- Branch-and-Bound nodes (BnB) - amount of partitions on the solution space

3 Method

- Formulate two IP models, one standard & with flow based optimizations for both MLBP(CC)
- Implement the models in CPLEX using C++
- Evaluate the models on a range of instances with varying sizes and complexities
- Compare and discuss the results

4 MLBP model

Capacity constraint

- $\sum_{i\in B^{(k-1)}} x_{ij}^k \cdot s(B_i^{(k-1)}) \le y_j^k \cdot w(B_j^k) \quad \forall k \in m, \forall j \in B^k$

Each item must be inserted into the bins of level 1

- $\sum_{i\in B^0} x_{ij}^1 = 1 \quad \forall j \in B^1$

Once a bin i is used at level (k-1), it also must be used at level k

- $\sum_{i\in B^{(k-1)}} x_{ij}^k = y_i^{(k-1)} \quad \forall k \in m, \forall j \in B^k$

Decision Variables: $y_j^k \in \{0,1\}$ $x_{ij}^k \in \{0,1\}$

Objective Function: $\min \sum_{k=1}^{m} \sum_{j\in B^k} y_j^k \cdot c(B_j^k)$

5 MLBP model with flow optimizations

The amount of incoming flow to the network equals to the number of items

- $\sum_{j\in B^1} f_{ij}^1 = 1, \forall i \in B^0$

The amount of outgoing flow from the network equals to the number of items

- $\sum_{j\in B^m} \sum_{i\in B^{(m-1)}} f_{ij}^m = n$

The amount of outgoing flow equals to the amount of incoming flow in every node

- $\sum_{a\in B^{(k-1)}} f_{aj}^k = \sum_{b\in B^{(k+1)}} f_{jb}^k \quad \forall k \in [2, ..., (m-1)], \forall j \in B^k$

$f_{ij}^1 \in \{0,1\}$ $f_{ij}^{>1} \in \mathbb{N}_{[0,n]}$

6 MLPBCC models

$z_{aj}^k \in \{0,1\}$

Populate z for level k=1

- $x_{aj}^1 = z_{aj}^1$ $\forall a \in B^0, \forall j \in B^1$

Populate z for levels k>1

- $(z_{ai}^{(k-1)} \wedge x_{ij}^k) \rightarrow z_{aj}^k$ $\forall k \in [2, m], \forall i \in B^{(k-1)}$ $\forall j \in B^k, \forall a \in B^0$

Conflict constraint

- $z_{aj}^k + z_{bj}^k \le y_j^k$ $\forall k \in m, \forall j \in B^k$ $\forall a \in B^0, \forall b \in B^0 : \text{conf}_{ab}$

	Model 1	Model 2
Same Ticks count	0	
Less Ticks count	208	42
Same BnB nodes count	36	
Less BnB nodes count	110	104

Table 4: Comparison of ticks and BnB nodes between the two MLBP models, with timeout $t = 600$ and the first 250 instances.

	Model 1	Model 2
Same Ticks count	359	
Less Ticks count	385	356
Same BnB nodes count	434	
Less BnB nodes count	335	331

Table 5: Comparison of ticks and BnB nodes between the two MLBPCC models, with timeout $t = 60$ and the first 1100 instances.

7 MLBP tick and BnB comparison

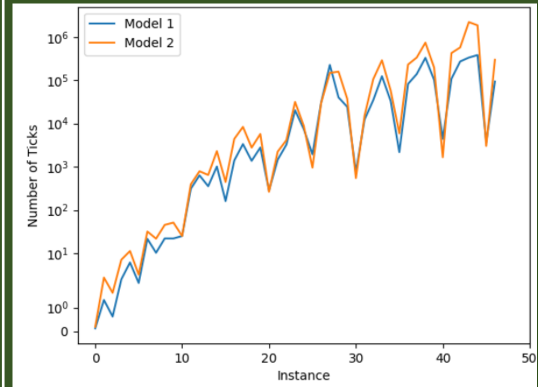

Figure 6: Visualisation of the tick comparison from Table 4 for the two MLBP models.

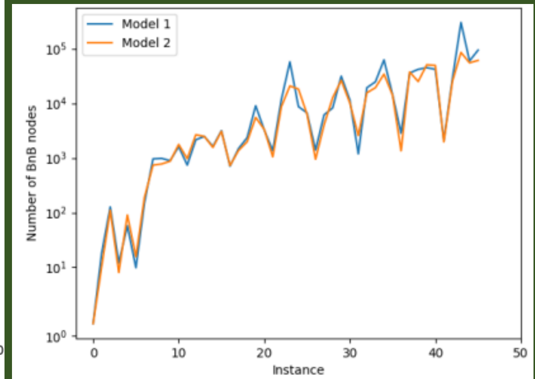

Figure 7: Visualisation of the BnB node comparison from Table 4 for the two MLBP models.

8 MLBPCC tick and BnB comparison

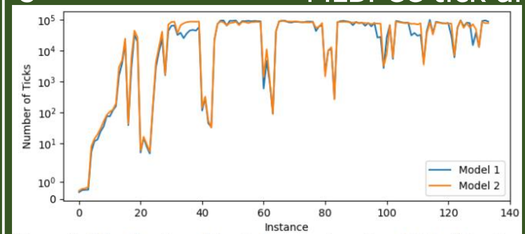

Figure 8: Visualisation of the tick comparison from Table 5 for the two MLBPCC models.

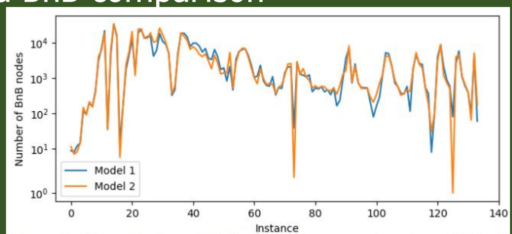

Figure 9: Visualisation of the BnB node comparison from Table 5 for the two MLBPCC models.

9 Limitations

- The experiments should be run on a more powerful machine, like a supercomputer
- More conclusive results for MLBPCC
- The flow based model can be further strengthened

10 Conclusion

- The initial hypothesis was wrong
- The standard model is faster
- The flow based model might be stronger, however the BnB comparison results are inconclusive

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