

1. Research Question

"How does incorporating various city plannings affect the formation of gang territories in a multispecies random walker model?"

- How do the model results compare to other multispecies random walker models that don't incorporate city planning?
- How do the model results compare to the actual gang territories in the replicated city planning?

2. Background

- Gang-related violence poses a significant challenge in numerous countries across the world [1].
- Existing random walker model - gang territories based on graffiti markings [2]
- Existing research on effects of terrain on other species' territories [3, 4]
- Gap in incorporating city layout to gang territories

3. Methodology

1. Extending Existing Model

a. A factor called "hardness" is added to the model that inversely affects an agent's probability of moving to a cell. The variable " α " represents the importance of this factor.

- The equations for graffiti production:

$$\xi_i(x, y, t + \delta t) = \xi_i(x, y, t) - (\lambda \cdot \delta t) \cdot \xi_i(x, y, t) + (\gamma \cdot \delta t) \cdot \rho_i(x, y, t)$$

Where ρ_i and ξ_i represents the agent density and graffiti density of a gang type "i" at a given location and time.

- The new equation for agent movement from cell (x_1, y_1) to cell (x_2, y_2) :

$$M_a(x_1 \rightarrow x_2, y_1 \rightarrow y_2, t) = \frac{e^{-\beta \xi_B(x_2, y_2, t) - \alpha h(x_2, y_2)}}{\sum_{(\tilde{x}, \tilde{y}) \sim (x_1, y_1)} e^{-\beta \xi_B(\tilde{x}, \tilde{y}, t) - \alpha h(x_2, y_2)}}$$

Where β represents the importance of the repellence of graffiti, and $(\tilde{x}, \tilde{y}) \sim (x_1, y_1)$ represents the neighbors of cell (x_1, y_1)

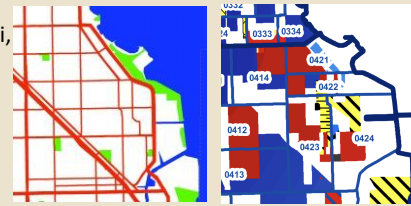


Figure 1: On the right the simplified and color-coded city layout map [5]. On the left the same area as the city layout from the Chicago PD gang territory map of 2022 [6].

2. Mathematical Analysis on a Simple Lattices

a. Observe effects of just changing the periodic boundaries to hard wall boundaries to make sure none of the observed effects are due to this change

b. Observe effects of changing α on the single vertical boundary via the order parameter :

$$\epsilon_2(t) = \left(\frac{1}{4N^2L} \right) \sum_{(x,y) \in H} (\rho_A(x, y-1, t) - \rho_B(x, y-1, t)) \cdot (\rho_A(x, y+1, t) - \rho_B(x, y+1, t))$$

Where H is the set of cells on the vertical boundary

c. Observe the effects of halving the lattice size since for large alpha values the model is assumed to behave as two separate lattices on either side of the boundary in the single vertical boundary runs.

d. Apply One-way ANOVA analysis to find statistical significance in changing α

3. Real World Application

a. Retrieve and edit a map of Chicago, where roads are dyed red, parks are dyed green, and water bodies are dyed blue such as in figure 1 on the left.

b. Use this as lattice and compare to real gang territory data seen in figure 1 on the right.

6. Conclusion and Future Work

Based on comparisons of the results against the results of preceding research, incorporating various elements of city layouts such as roads, lakes, and parks significantly affect the shapes and size of gang territories. And by adjusting the permeability of such boundaries within our model, we can bring the model closer to real world scenarios where gang territories do show resemblance to the terrain of the neighborhood they reside in as well as the real gang territories found in the same neighborhood. It could be interesting to see the results with more than two gang types, or with topographic terrains on top of city layout.

4. Mathematical Analysis Results and Discussion

- No effects to the shape and size of territories were observed from changing the lattice boundaries or size.
- As seen in Figure 2, for alpha values between 0 and 0.4 we observe a clear reduction in the order parameter value however after 0.4 the lines begin to overlap, and it is hard to make conclusions.
- The box and whisker plots are made as seen in Figure 3. Here it seems like increasing the alpha value further then 0.5 poses no change in the order parameter value.

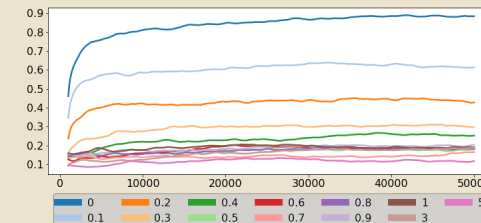


Figure 2: Order parameter vs. time for various α

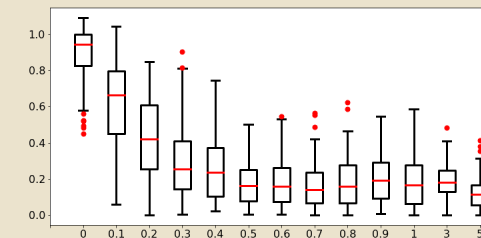


Figure 3: Order parameter vs. α in the form of box and whisker plots

- Table 1 showcases the results of ANOVA analysis. A p value less than 0.05 indicates a statistically significant difference between the final order parameter values of the two α values [7]. α values between 0 and 0.4 all show a statistical significance to all other values with some minor exceptions. There appears to be a critical α value between 0.4 and 0.5 before which there is a statistically significant difference for differing α values.

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	3	5
0	X	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	X	0	0	0	0	0	0	0	0	0	0	0
0.2	0	0	X	0	0	0	0	0	0	0	0	0	0
0.3	0	0	0	X	0.111	0	0	0	0	0	0	0	0
0.4	0	0	0	0.111	X	0.001	0.001	0	0.002	0.018	0.003	0.001	0
0.5	0	0	0	0	0.001	X	0.932	0.672	0.841	0.24	0.66	0.53	0.001
0.6	0	0	0	0	0.001	0.932	X	0.74	0.78	0.214	0.605	0.477	0.001
0.7	0	0	0	0	0	0.672	0.74	X	0.549	0.117	0.399	0.278	0.004
0.8	0	0	0	0	0.002	0.841	0.78	0.549	X	0.354	0.822	0.711	0.001
0.9	0	0	0	0	0.018	0.24	0.214	0.0117	0.354	X	0.476	0.489	0
1	0	0	0	0	0.003	0.66	0.605	0.399	0.822	0.476	X	0.907	0
3	0	0	0	0	0.001	0.53	0.477	0.278	0.711	0.489	0.907	X	0
5	0	0	0	0	0	0.001	0.001	0.004	0.001	0	0	0	X

Table 1: p values from one-way ANOVA analysis on all α pair's final order parameter values

5. Real World Application Results and Discussion

- The model was run on Chicago map with α values of 0.35 and 0.65 to observe the effects below and above the critical α threshold. Figure 4 showcases the results of these runs with α of 0.35 on the left and α of 0.65 on the right.
- With both images, we can see the roads, lake and parks indeed show a lack of gang presence which matches the real territories found in Figure 1.

- For α of 0.35, while the roads and parks have some effect, there are many territories that don't fully cover a neighborhood yet span across multiple rectangular areas split via road.
- However, for α of 0.65 most territories match the areas between the

obstacles very well. These rigid, rectangular shapes match the real territories seen in figure 1 on the right.

- These results match a critical α value between the expected range.

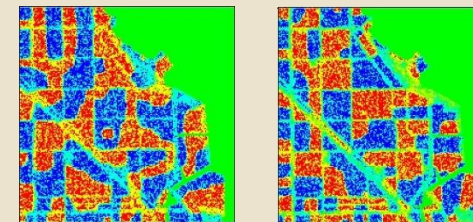


Figure 4: Model run on a part of Chicago's city layout with α values of 0.35 and 0.65 on the left and right respectively.

Citations

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