Modelling Finite State Automata in Agda

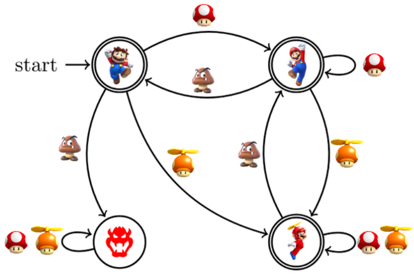
Noky Soekarman, Supervisor: Bohdan Liesnikov, Responsible Professor: Jesper Cockx

Contact: N.P.Soekarman@student.tudelft.nl

1. Introduction

Finite State Automata (FSA)

- Input Alphabet
- Set of states
- For every element in the alphabet there exists a transition to a next state (DFA).
- For a nondeterministic version there are zero or more transitions for each element in the alphabet.
- See the example with the Mario, whenever he takes a powerup or bumps into a goomba, he moves to another state.



Agda

- Functional programming language
  - Programs are constructed by applying and composing functions.
- Dependently typed
  - Dependent types are introduced by having families of types indexed by objects in another type.
- Total language
  - Program e of type T will always terminate with a value in T.

Coinduction

- Coinduction can be used to represent infinite or cyclic structures.
- To get values from these structures, observers and destructors are used.
- Two coinductive types are bisimilar if they have the same behaviour.

2. Research Question:

What are different ways to model Finite State Automata in Agda in a coinductive way and how to prove equivalence for them?

3. Methods

- Research what FSA are and when they are equivalent.
- Look into past work to see what has been done already.
- Implement different encodings of FSA.
- Try to prove equivalence for the various encodings.
- Compare the encodings.
- Suggest possible improvements for Agda to make it easier to use or implement FSA using coinduction.

4. Results

- Two fully working DFA encodings, both in Guarded and Musical style.
- A notion of equivalence for all the DFA encodings.
- An NFA encoding which can run input.

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5. Agda Issues

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  - This made learning and understanding coinduction in Agda take more time.
- Error messages are not clear enough in Agda.
  - As a results, you needed to spend quite some time figuring out what was actually wrong.
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6. Limitations and future work

- The encodings encode a single state, one needs to be mindful of this when using the encodings.
- Some encodings do not force you to create a set of states exactly according to the official definition.
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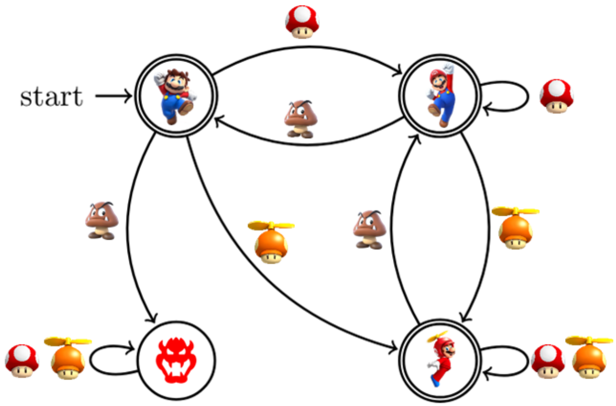

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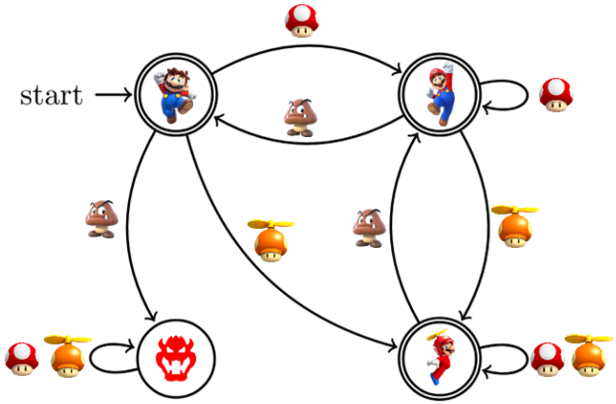

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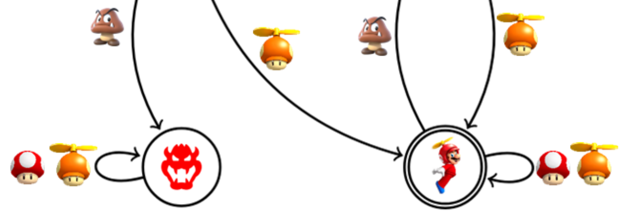

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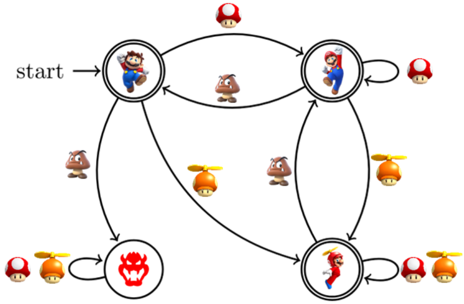

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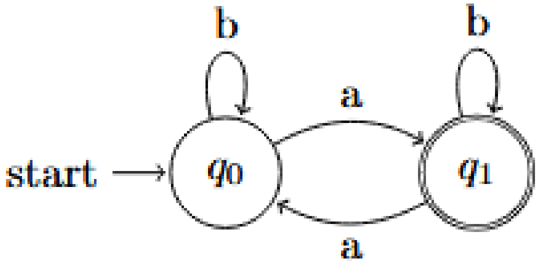
Example proof



The proof for this in Agda:

```
mutual
  q0_bisim_q0 : q0 ~ q0
  q0_bisim_q0 .accept = refl
  q0_bisim_q0 .transition a = q1_bisim_q1
  q0_bisim_q0 .transition b = q0_bisim_q0

  q1_bisim_q1 : q1 ~ q1
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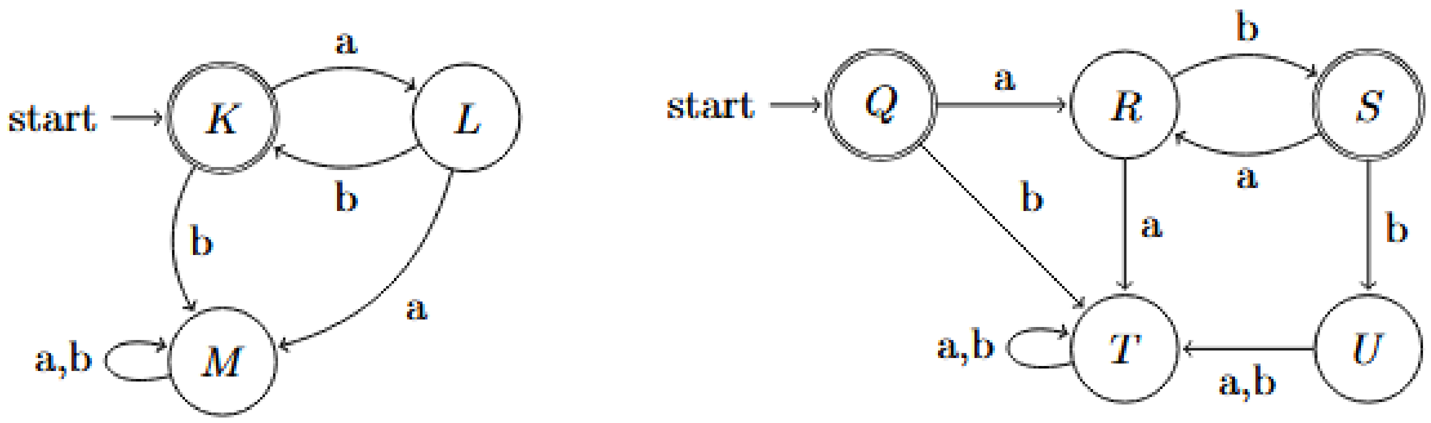

(a) DFA D_1

(b) DFA D_2

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```

```
k_bisim_q .accept = refl
k_bisim_q .transition a = l_bisim_r
k_bisim_q .transition b = m_bisim_t
```

Bad error message

```
q_reject !=
if
Relation.Nullary.Decidable.Core.isYes
(Relation.Nullary.Decidable.Core.map' Data.Char.Properties.≈⇒≡
 Data.Char.Properties.≈-reflexive
 (Relation.Nullary.Decidable.Core.map'
  (Data.Nat.Properties.≡ᵇ⇒≡ 97 (toN c))
  (Data.Nat.Properties.≡⇒≡ᵇ 97 (toN c))
  ((97 Agda.Builtin.Nat.== toN c)
   Relation.Nullary.Decidable.Core.because
   Relation.Nullary.Reflects.T-reflects
   (97 Agda.Builtin.Nat.== toN c))))
then q1 else getFromList c (('b' , q0) :: [])
of type DFAState
when checking that the expression q_reject_bisim_q_reject has type
getFromList c (q0 .transition) ~ getFromList c (q0 .transition)
```

Termination checker cheat

```
{-# TERMINATING #-}
getReachableStates : NFAState → List NFAState → List NFAState
getReachableStates currentState visitedStates = if stateInList currentState visitedStates then [ ]
  else
    let
      newVisitedStates = currentState :: visitedStates
      epsilonStates = getEpsilonStates (currentState .transition)
    in
      currentState :: concatMap (λ s → getReachableStates s newVisitedStates) epsilonStates

getUniqueStates : List NFAState → List NFAState
getUniqueStates [ ] = [ ]
getUniqueStates (x :: xs) = if stateInList x xs then getUniqueStates xs else x :: getUniqueStates xs

runNFA : NFAState → List Char → Bool
runNFA currentState [ ] = currentState .isAccepting
runNFA currentState (c :: cs) =
  let
    reachable = getUniqueStates (getReachableStates currentState [ ])
    nextStates = getUniqueStates (concatMap (λ s → getFromListWithoutEpsilon c (s .transition)) reachable)
    finalStates = getUniqueStates (concatMap (λ s → getReachableStates s [ ]) nextStates)
  in
    any (λ s → runNFA s cs) finalStates
```