# Learning Reduced-Order Mappings between Functions

## An Investigation of Suitable Inputs and Outputs

Bo Bakker[†][*]

[*]EEMCS, TU Delft, [†]`b.bakker-1@student.tudelft.nl`

## 1. Introduction

- Many tasks in science and engineering rely on solving PDEs
- Data-driven PDE solvers → Leverage previous solutions
- PCA-based NN solvers [1] exist, but what are the limitations?

*"What are the limitations on the types of inputs and outputs PCA-NN solvers can provide adequate solutions for?"*

## 2. Methodology

Creating the Dataset:

1. Generate Gaussian Random Fields (GRFs) [2], 27 sets
    - Covariance Function (Gaussian, Exp., Sep. Exp.)
    - Correlation Length (0.15, 0.1, 0.05)
    - Variance (1, 10, 100)
2. Use Finite Element PDE solver, create outputs from GRFs
    - Poisson's Equation, Heat Equation → 54 sets total

Training and Testing the Model(s):

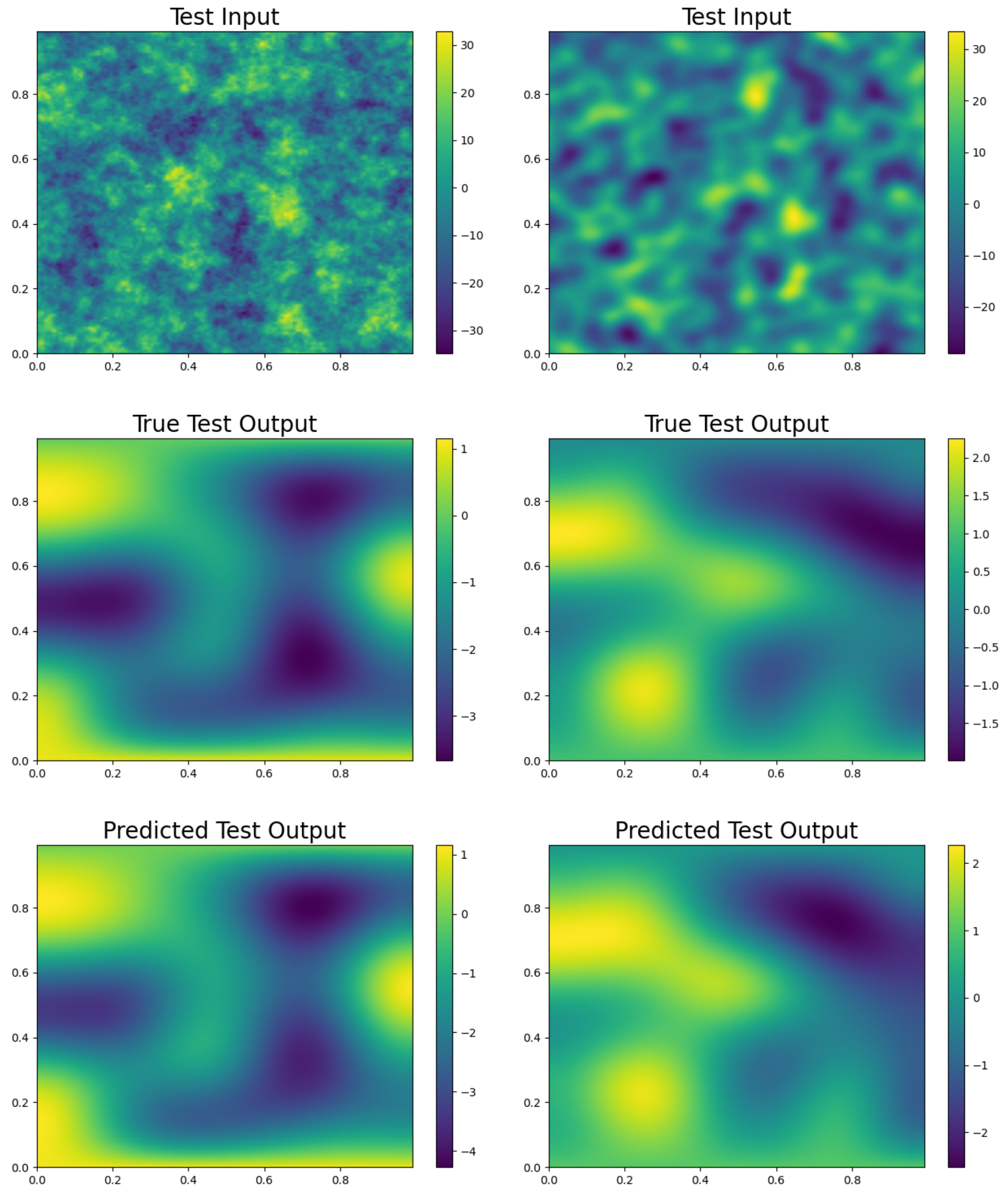
1. Split data into training and testing data
    - 2000 input-output pairs, 50-50 split, 1000 each
2. Apply PCA to training data (both input and output)
    - Number of PCA components → Accuracy of 99%
3. Train fully connected neural network on training data
    - Input and output layers → Number of PCA components
4. Evaluate performance on both training and testing data
    - Compare Relative MSE between sets of parameters

## References

[1] K. Bhattacharya et al, "Model reduction and neural networks for parametric pdes," 2021.

[2] G. J. Lord et al, *An Introduction to Computational Stochastic PDEs*, Ch. 7, 2014.

## PCA-NN (Heat Equation)



(a) Exponential (b) Gaussian

Inputs and Outputs for the Heat Equation, with a correlation length of 0.05 and a variance of 100

## 3. Results

- Unexpected results
    - Different patterns for both equations
    - "Rougher" inputs usually perform better

Poisson's Equation:

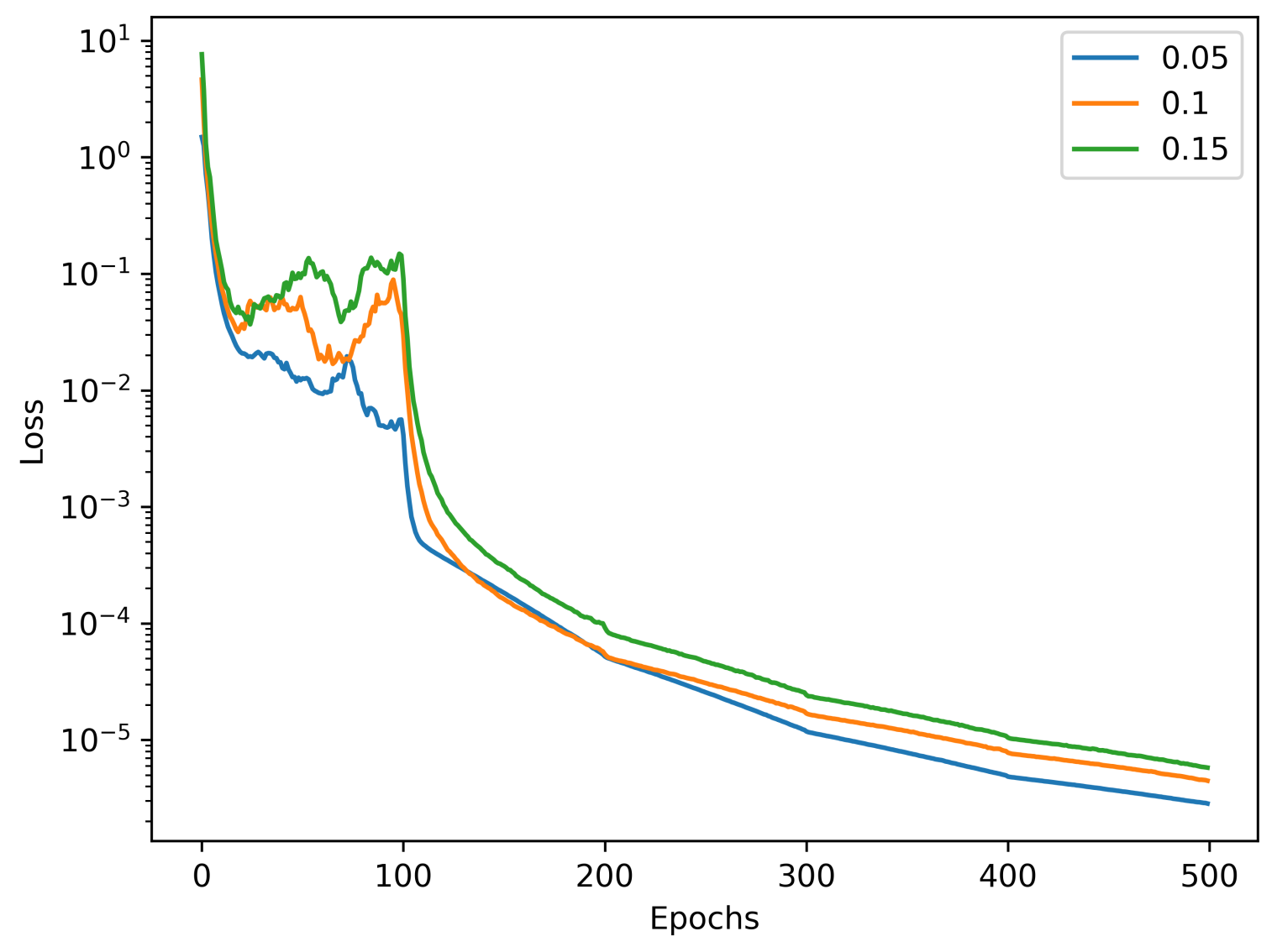
- Gaussian covariance:
    - Equal performance for smaller correlation lengths
- Exponential and Separable Exponential covariance:
    - Better performance for smaller correlation lengths
- Number of PCA components for output varies significantly

Heat Equation:

- Gaussian covariance:
    - Worse performance for smaller correlation lengths
- Exponential and Separable Exponential covariance:
    - Better performance for smaller correlation lengths
- Number of PCA components for output stays fairly constant

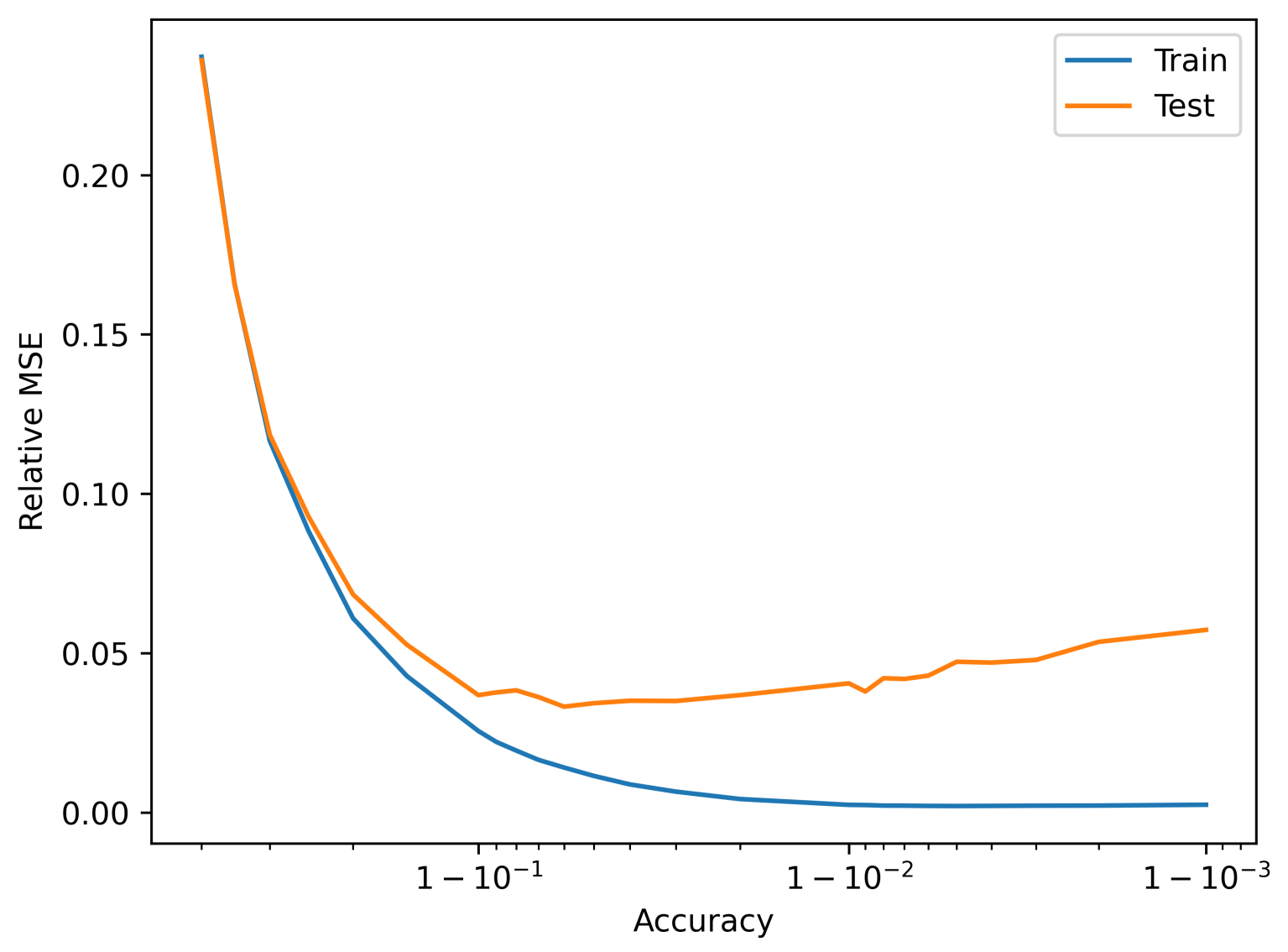
Smaller variance results in better performance across the board

## Training Loss



Training loss for Poisson's Equation, with Gaussian covariance, and a variance of 1

## MSE vs Accuracy



Training and testing error when varying the accuracy percentage

## 4. Conclusion

From the results, we can conclude the following:

- Patterns were discovered in testing error when varying parameters
- Error may vary significantly when changing correlation length
- Performance is adequate for all inputs (error always below 0.35)
- PCA-NN is well suited for solving these types of problems

## 5. Limitations & Future Work

Difficult to extrapolate from these results → Further research needed:

- See if the patterns hold for other covariance functions
- See if the patterns hold for other Elliptic and Parabolic equations
- Try to discover similar patterns for Hyperbolic equations
- Retry with the number of PCA components held constant