Multi Species Turing Patterns in 3D

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1 Background

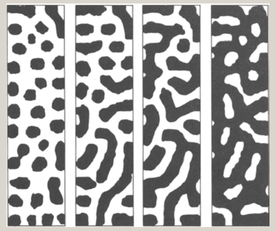
Alan M. Turing [1] 1952

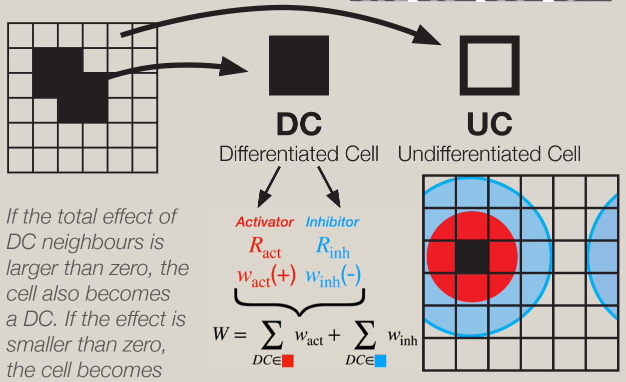
Morphogenesis, described through a continuous reaction-diffusion process between two chemicals. *"Turing patterns"*

David A. Young [2] 1984

Discretisation of Turing's model that allows computer simulation.

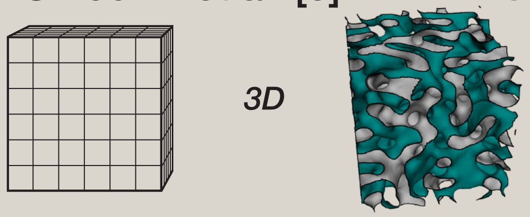
"Cellular Automaton"





If the total effect of DC neighbours is larger than zero, the cell also becomes a DC. If the effect is smaller than zero, the cell becomes a UC.

M. Skrodzki et al. [3] 2017

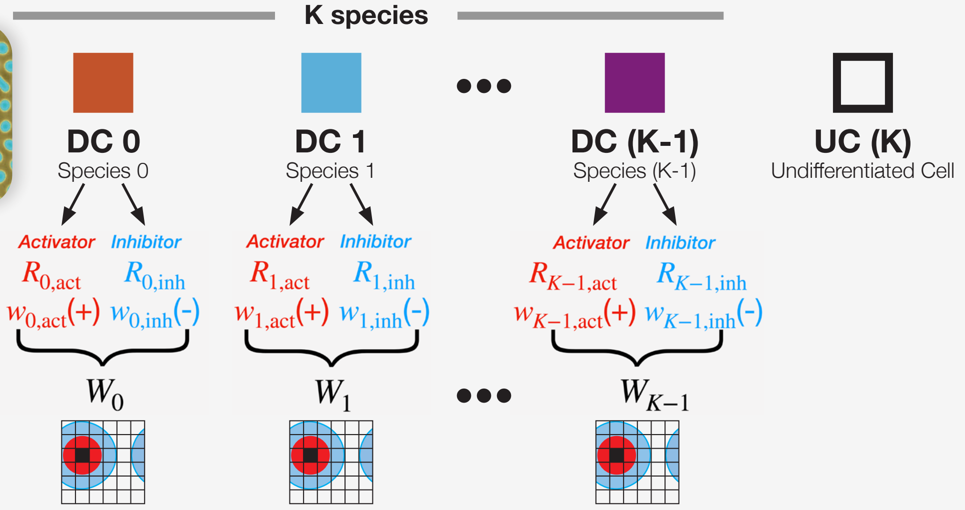


Research Objective 2023

1. How can an existing three-dimensional variant of Young's model be generalised to produce **multi-species Turing-like structures**?
2. To what extent can the formation of such structures be quantified through an **order-parameter**?

2 Methodology

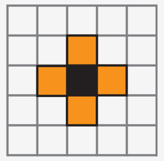
1 Multi Species Extension



When updating the state of the cellular automaton, for each cell we apply the following **individual update rules**.

1. If the maximum W is above zero, the cell gets assigned the corresponding species.
2. If the maximum W is below zero, the cell gets assigned UC.
3. Else, the cell's state doesn't change.

2 Order Parameter $\vec{E}$



In 3D, there are six direct neighbours

$$\vec{E} = [\epsilon_0, \epsilon_1, \ldots \epsilon_{K-1}, \epsilon_K \equiv \epsilon_{\text{undif}}]$$

Sum over all cells | Sum over direct neighbours

$$\epsilon_j(t) = \frac{1}{6L^3} \left| \sum_{c\in D} \sum_{\tilde{c}\in\mathcal{N}(c)} \left(\sigma_j(c,t)\cdot\sigma_j(\tilde{c},t)\right) \right|$$

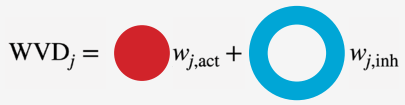
Size of the domain

Order parameter inspired by [5]

Intuition for $\sigma_j(c,t)$
If our neighbour is also (not) of species j: **+1**
If I am species j, but my neighbour is not: **-1**

$$\sigma_j(c,t) = \begin{cases} 1 & c(t) = j \\ -1 & \text{otherwise} \end{cases}$$

3 Weighted Volume Difference

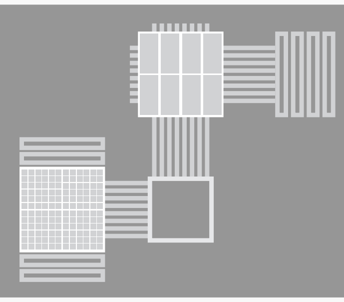


$$\text{WVD}_j = \frac{4}{3}\pi\left(R^3_{j,\text{act}} w_{j,\text{act}} + (R^3_{j,\text{inh}} - R^3_{j,\text{act}}) w_{j,\text{inh}}\right)$$

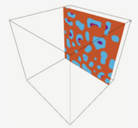
"Are the chemicals balanced?"

4 GPU Acceleration

A big part of this research was devoted to accelerating the simulations using the Graphics Processing Unit (GPU). Researching the above would not have been possible without this acceleration.

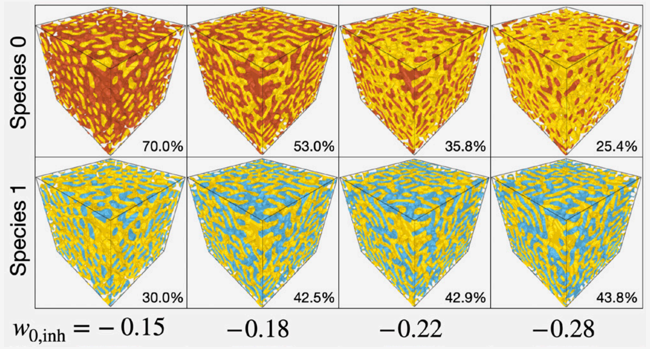


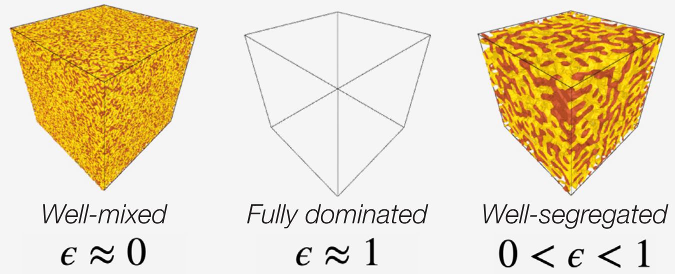
3 Results

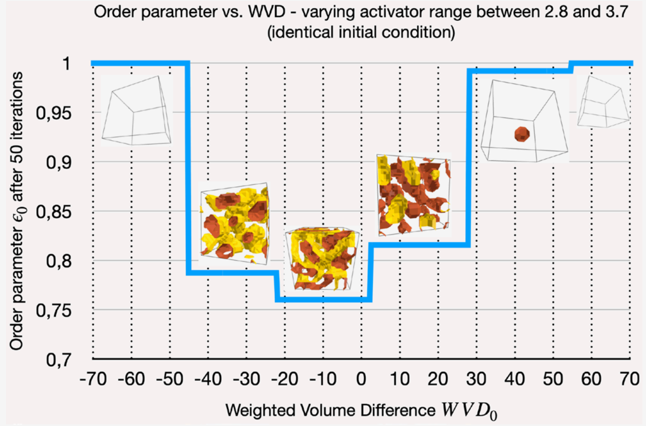


Debugging visualisation. Show one intersection at a time. Not sufficient for revealing 3D patterns.

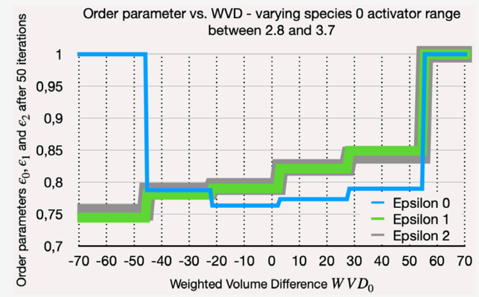
Varying the chemical configuration of one species has seemingly little impact on pattern formation in other species, as long as the configurations remain *balanced*.







The order parameter reacts to changes in the structures that form, however further research is required before it can be used to classify shapes.



The order parameter of one species changes upon change of another species' configuration.

4 Conclusion

1. Turing patterns appear in the multi-species case.
2. It seems that pattern development in one species has little dependency on chemical configuration of another species. This must be further investigated.
3. Order parameter can distinguish between well-mixed, well-segregated and fully dominated states.
4. Order parameter reacts to changes in structures, but further research is required to investigate its usefullness in classification of shapes.

5 Limitations

1. Even though Turing patterns have been connected to patterns in biology and other fields, the results presented here have no direct relation with those fields.
2. Simulations only run on macOS.

6 Opportunities

1. Classification of shapes in multi-species case.
2. Discovering the role of the order parameter in this classification.
3. When types of shapes are quantified, investigate influence of one species on the other.
4. How does kernel shape influence pattern formation?
5. How does the initial condition affect pattern formation?
6. What new patterns from the world around us can we recreate with this multi-species extension?

References

[1] A. M. Turing - *The Chemical Basis of Morphogenesis* (1952)
[2] D. A. Young - *A Local Activator-Inhibitor Model of Vertebrate Skin Patterns* (1984)
[3] M. Skrodzki, K. Polthier - *Turing-like patterns revisited: A peek into the third dimension* (2020)
[4] S. Kondo, T. Miura - *Reaction-diffusion model as a frame- work for understanding biological pattern formation* (2010)
[5] D. P. Barajas - *Implementation and Evaluation of an Order Parameter for the Reaction-Diffusion Model in a Cellular Automaton. in prepration (2023)*

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