Algorithms for Resource Exchange and Link Formation in Decentralized Networks

Strategic link formation, link costs, discounting, degradation, and equilibrium in dynamic exchange networks

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1. BACKGROUND

- ▶ Agents exchange resources through network links and can create additional links with costs that have a lifetime.
- ▶ A poorly connected agent would want to connect to other peers to receive resources.

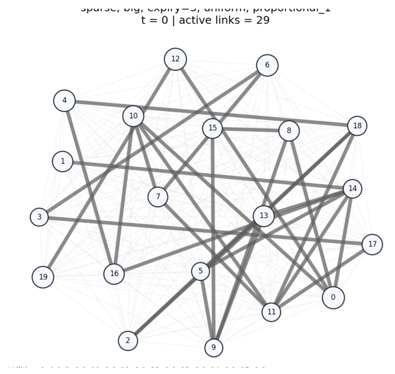
2. RESEARCH QUESTION

How does strategic, costly link formation affect the welfare, fairness, and stability of decentralized resource-exchange networks?

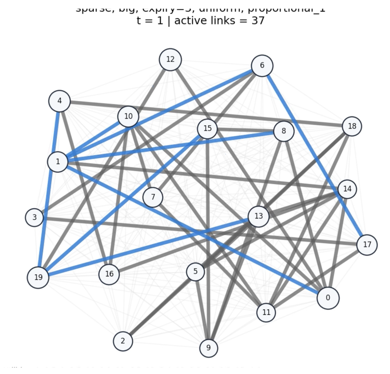
3. MODEL SETUP AND ASSUMPTIONS

- ▶ N agents, each produces $a_i(t) > 0$ resources in each slot, shares with neighbors x_{ij}, creates links
- ▶ Created links have costs and a lifetime
- ▶ Degradation $\beta \in (0,1)$ and Discount factor $\delta \in (0,1)$.
- ▶ Creates link if estimated benefits with link - without link is bigger than costs
- ▶ Estimated benefits don't include second-order effects

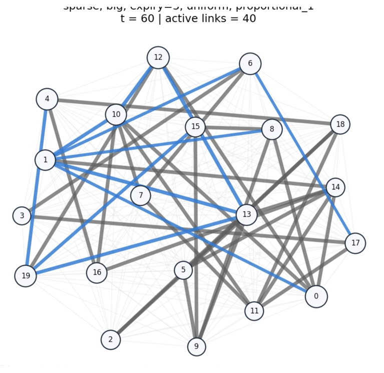
4. SIMULATION SNAPSHOT: GRAPH EVOLUTION



At $t = 0$:
29 active links



At $t = 1$:
37 active links



At $t = 60$:
40 active links

5. KEY DEFINITIONS

Effective received resource

Resource received after link quality is applied.

$$r_i(t) = \sum_{j \in N_i(t)} q_{ij}(t) x_{ji}(t)$$

Intuition: Received from neighbors times quality(influenced by degradation)

Link creation rule

Create a new link $\{i, j\}$.

$$\sum_{\tau=t+1}^{t+T_{ij}} \delta^{\tau-t} \left(r_i^{+ij}(\tau) - r_i^{-ij}(\tau)\right) \geq c_{ij}$$

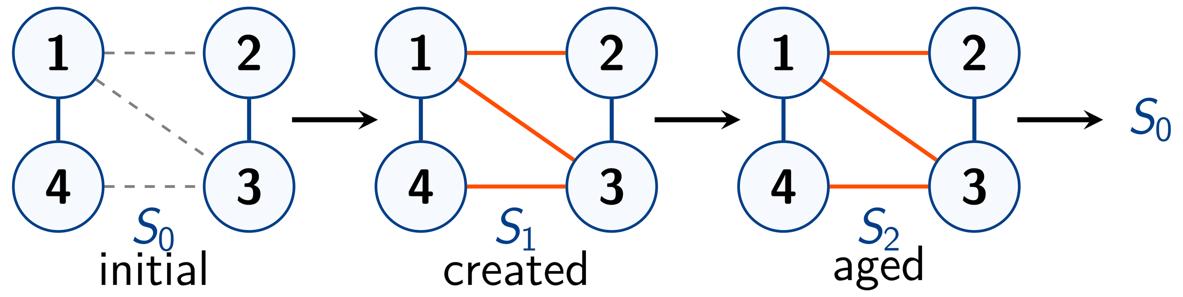
Intuition: Create only if the discounted future gain covers the creation cost.

6. STABLE STATES / CYCLES

State: S_t because no classical equilibrium.

Improving cycle: instead of reaching one fixed state, the system may repeat:

$$S_1 \to S_2 \to \cdots \to S_p \to S_1.$$



7. EGALITARIAN STRATEGY

Idea: under egalitarian allocation, an agent splits its resource equally among all active neighbours.

$$x_{ij}^{eg}(t) = \frac{a_i(t)}{d_i(t)}$$

Observations for the egalitarian strategy

- ▶ The egalitarian strategy reaches a cycle because the effective number of states is finite.
- ▶ Sparse candidate neighbours are preferred, since the expected benefit is larger when $d_j(t)$ is smaller.
- ▶ The expected benefit of a new link is highest at the first time slot.

8. PROPORTIONAL STRATEGY

Idea: under proportional allocation, an agent sends more resource to neighbours from which it received more effective resource in the previous time slot.

$$x_{ij}(t+1) = a_i(t+1) \frac{q_{ij}(t) x_{ji}(t)}{\sum_{k \in N_i(t)} q_{ik}(t) x_{ki}(t)}$$

Two proportional strategies are used

- ▶ **Proportional 1:** newly created links receive initial equal allocation
- ▶ **Proportional 2:** newly created links receive a temporary exploration boost

ε-cycle for proportional strategies

- ▶ Allocation values are continuous.
- ▶ Cycles are detected approximately.

$$|S_{t+p} - S_t| < \varepsilon$$

9. SIMULATION METHODOLOGY

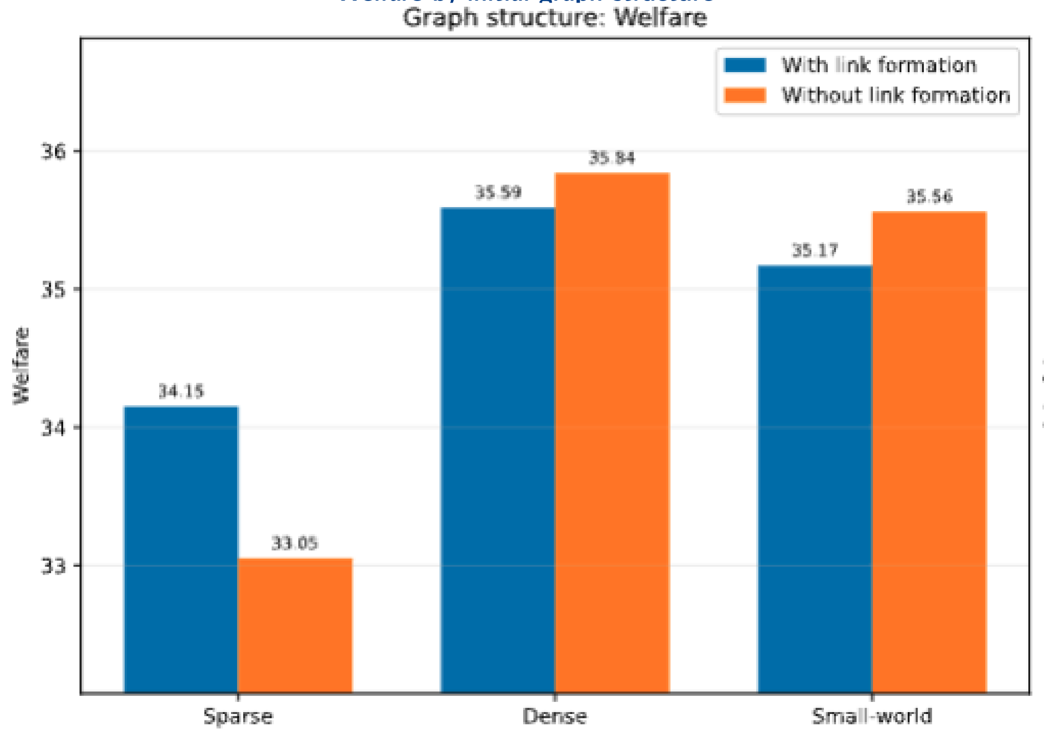
Simulation order in each time slot

1 Graph at time t → **2** Allocate resources → **3** Compute received resource → **4** Renew/remove links → **5** Create links check cycles

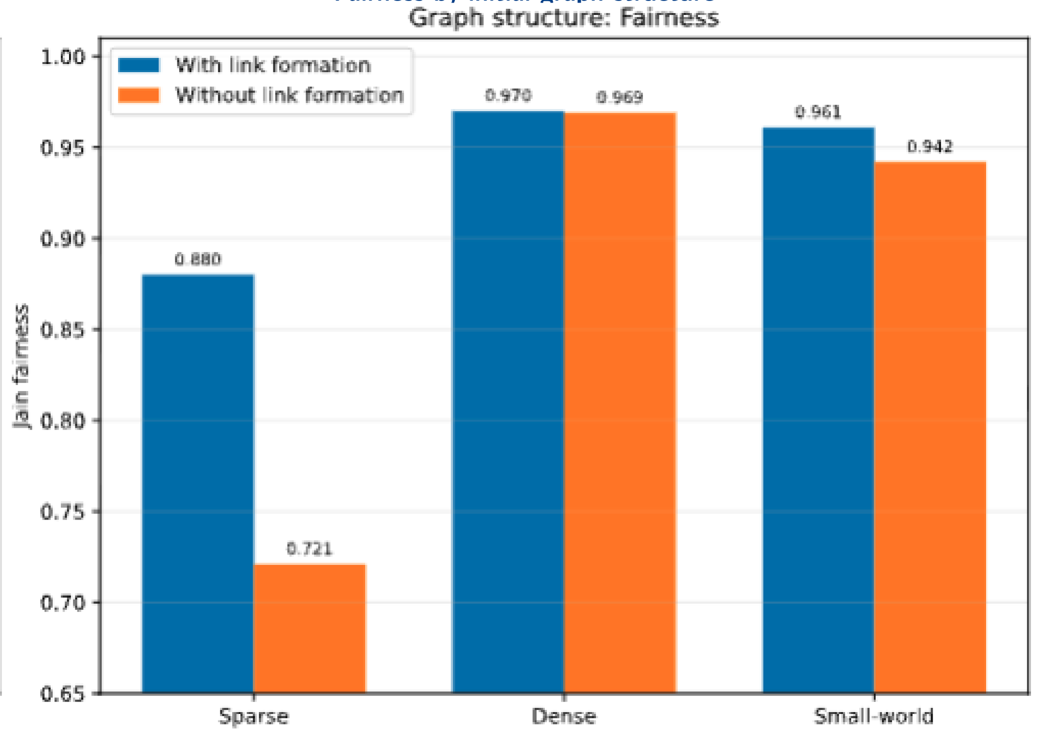
10. FAIRNESS AND WELFARE RESULTS

Metrics: Jain fairness Eisenberg–Gale welfare

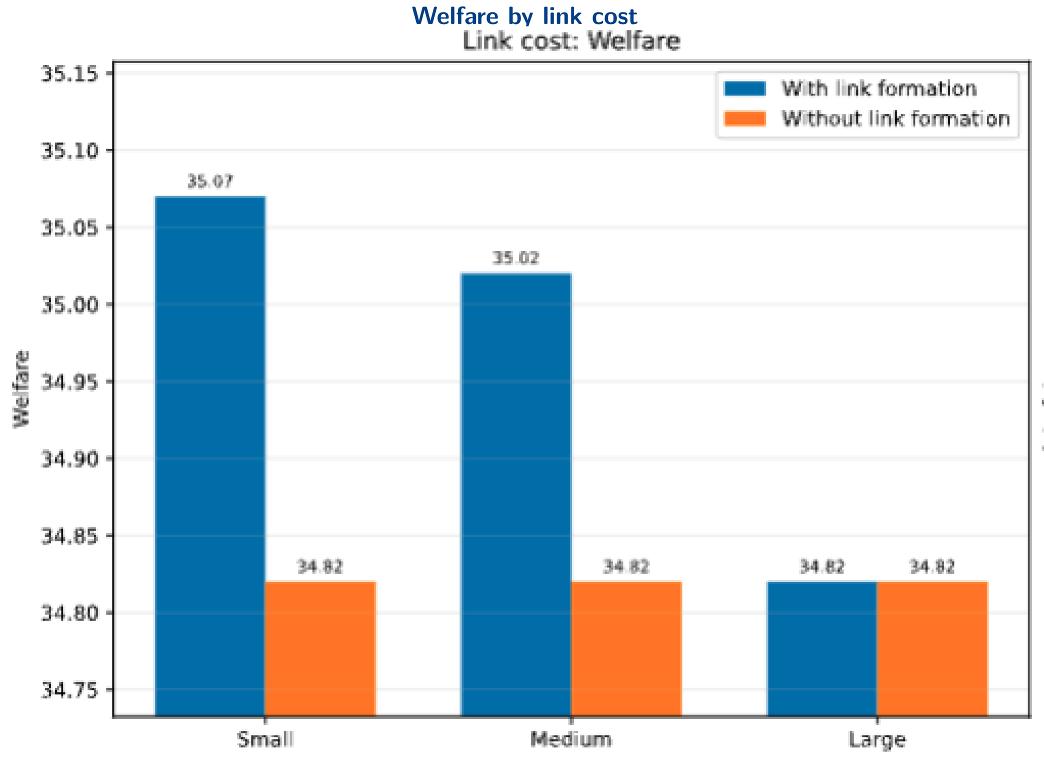
Welfare by initial graph structure



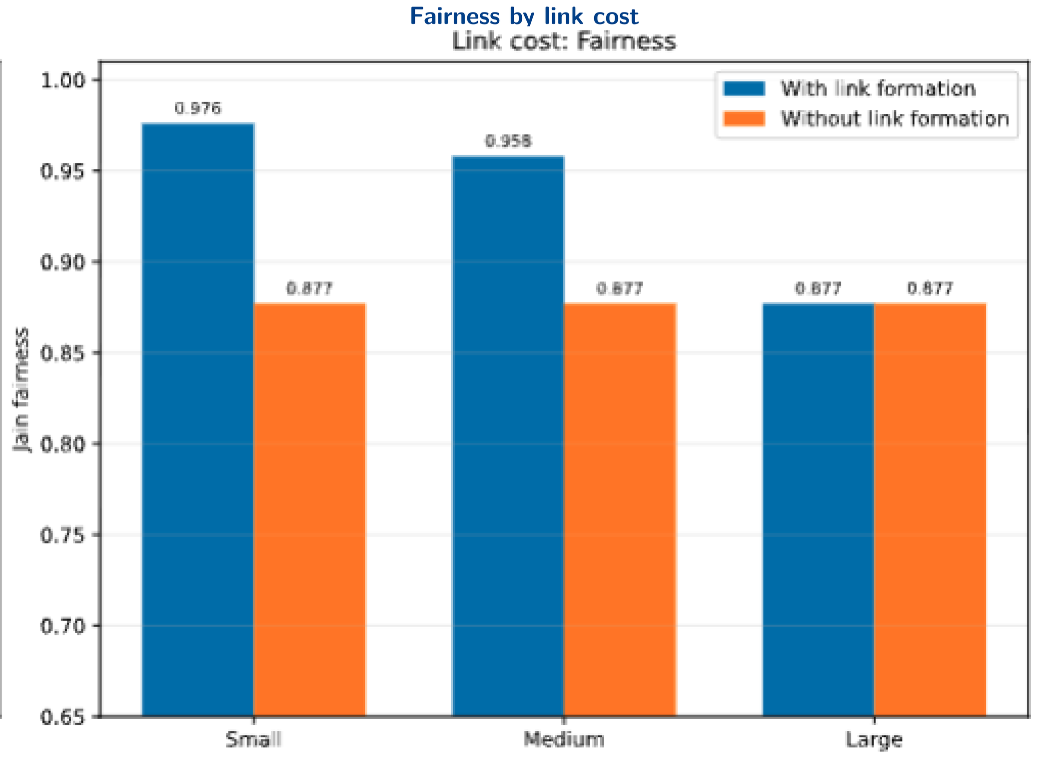
Fairness by initial graph structure



Welfare by link cost



Fairness by link cost



11. CONCLUSIONS

- ▶ Link formation can enter a cycle of states instead of an equilibrium.
- ▶ Strategic link formation can improve welfare and fairness, especially when the initial graph is sparse.
- ▶ When link costs are high, agents create fewer links and the system behaves more like the fixed-network case.
- ▶ More links are not always better: additional links can increase complexity without improving fairness or welfare.