A CELLULAR AUTOMATON FOR MODELING TERRITORIES

Contact:
Maarten van Bijsterveldt
M.P.vanBijsterveldt@student.tudelft.nl
EEMCS, TU Delft
Supervisors:
Martin Skrodzki (m.skrodzki@tudelft.nl)
Alethea Barbaro (A.B.T.Barbaro@tudelft.nl)

1. Background

Based on random-walk (RW) model [1]:

- Agents of two groups walking on a lattice, leaving markings behind.
- The direction they choose to walk in next is random, only influenced by their preference to avoid rival territorial markings.
- These markings decay over time. Example results of this model are shown below in Figure 1.

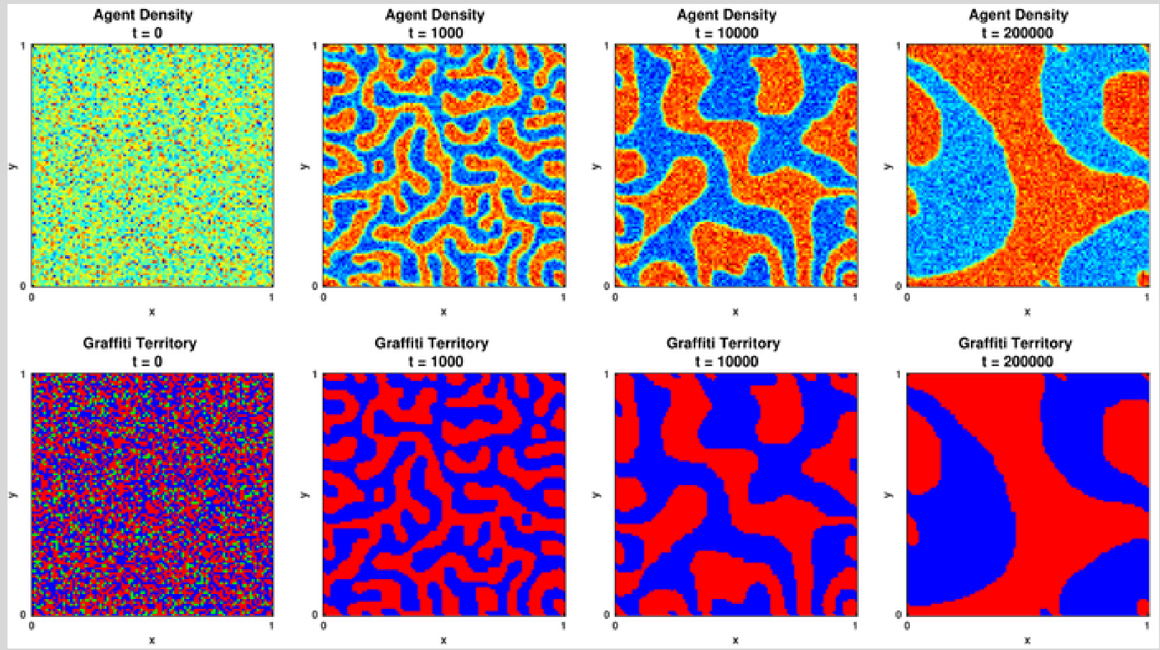

Fig. 1: Example results of the random-walk model.

Cellular automata [2]:

- A cellular automaton has cells on a grid with a state.
- Each time step, they can change to a new state. The new state depends on their neighborhood according to some rule.

2. Research Question

To investigate if a cellular automaton using only markings could give the same results as the random-walk model, the research question is:

"Does a cellular automaton for simulating territories, using only territorial markings, get similar outcomes as a random-walk algorithm?"

References

[1] A. Alsenafi and A. B. T. Barbaro, “A convection–diffusion model for gang territoriality,” Physica A: Statistical Mechanics and its Applications, vol. 510, pp. 765–786, 2018, doi: https://doi.org/10.1016/j.physa.2018.07.004.

[2] S. Wolfram, ‘Statistical mechanics of cellular automata’, Reviews of Modern Physics, vol. 55, no. 3, pp. 601–644, 1983.

3. Methodology

- Each cell has a state consisting of two continuous values which represent the markings of each group. This value changes based on a decay rate λ, the neighboring values, an avoidance β, and a neighbor influence parameter α.
- The update rule, where $\xi_i(x, y, t)$ is the markings of group i at cell (x,y) at time t, for both values is then:

$$\xi_i(x,y,t+1) = \xi_i(x,y,t) - \lambda\xi_i(x,y,t) + \alpha e^{-\beta\xi_j(x,y,t)} \cdot \frac{\sum_{(\tilde{x},\tilde{y})\sim(x,y)} \xi_i(\tilde{x},\tilde{y},t)}{4}$$

j is the other gang, and $(\tilde{x},\tilde{y})\sim(x,y)$ represents the neighbors of (x, y).

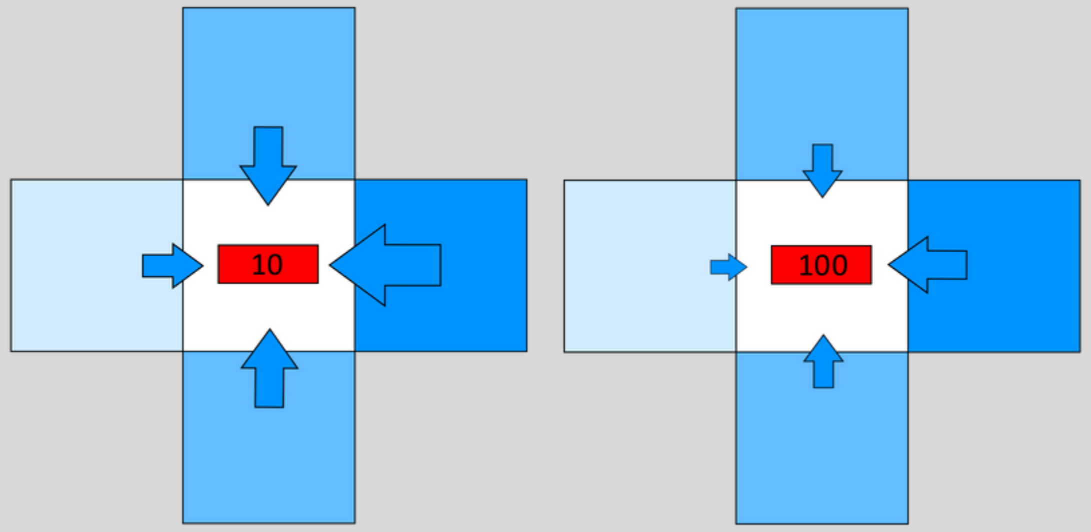

Fig. 2: Illustration of how the model works. Surrounding cells with blue markings have influence. This can be lessened by having more red markings in the current cell.

- An order parameter is used to analyze the results.
- It gets larger when neighboring cells have the same group as the majority and smaller when different groups have it.
- The magnitude depends on the ratio between both groups.

$$\varepsilon(t) = \left(\frac{1}{2L}\right)^2 \left| \sum_{(x,y)\in S} \sum_{(\tilde{x},\tilde{y})\sim(x,y)} \frac{\xi_A(x,y,t) - \xi_B(x,y,t)}{\xi_A(x,y,t) + \xi_B(x,y,t)} \cdot \frac{\xi_A(\tilde{x},\tilde{y},t) - \xi_B(\tilde{x},\tilde{y},t)}{\xi_A(\tilde{x},\tilde{y},t) + \xi_B(\tilde{x},\tilde{y},t)} \right|$$

where L is the lattice size and S the set of all cells.

- The order parameter can be plotted against time or parameters to see how the model behaves.

5. Conclusion

- Depending on the parameters, end results can be similar to the random-walk model.
- However, they evolve in different ways and the effect of parameters is different
- Parameters can make outcome vary drastically.
- This model could be more useful for different processes, such as spread of languages or religion.

4. Results

- A low avoidance (β = 0.00001) evolves into a well-mixed state (Figure 3).

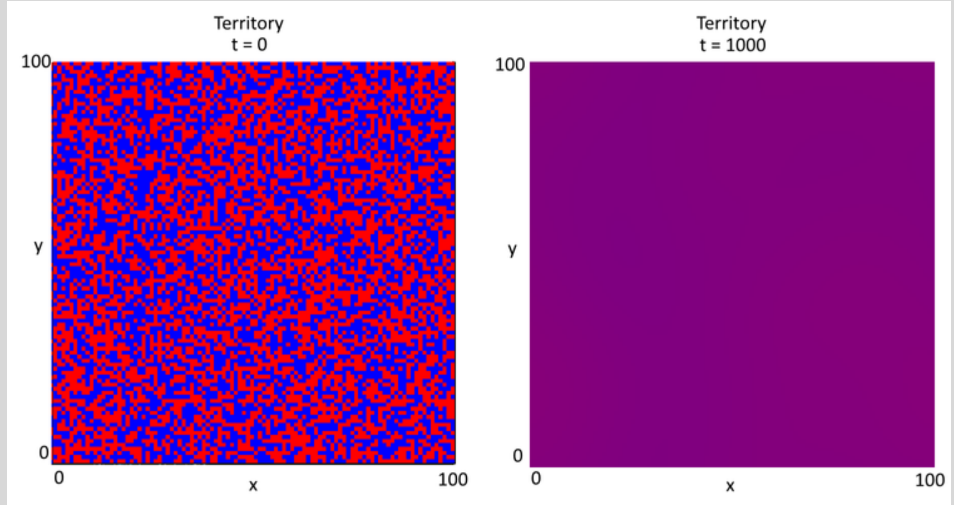

Fig. 3: Example of a well-mixed state.

- A high avoidance (β = 2) evolves into a semi-stable segregated state (Figure 4).
- Stays stable for a long time but eventually one takes over.

- By plotting the order parameter against β, it is visible how it changes (Figure 5).
- Between certain values, the order parameter rises from low to high.

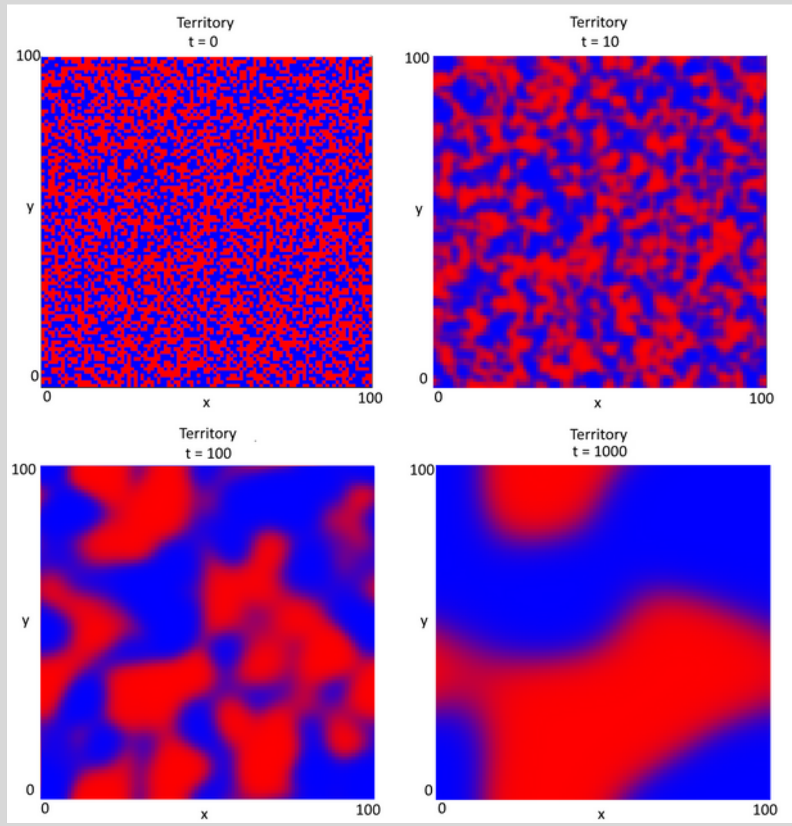

Fig. 4: Example of a semi-stable segregated state.

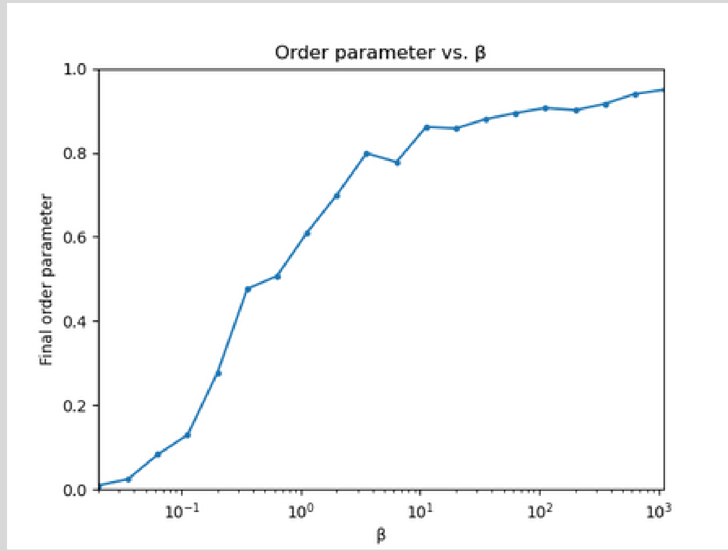

Fig. 5: The final order parameter plottet against β. Each point is the average of 5 simulations.

- By changing λ a bit (from 0.5 to 0.48) the segregated state has clear boundaries and large connected territories.
- This does however remove the mixed state: instead, it mixes but not completely, and then creates clear territories.

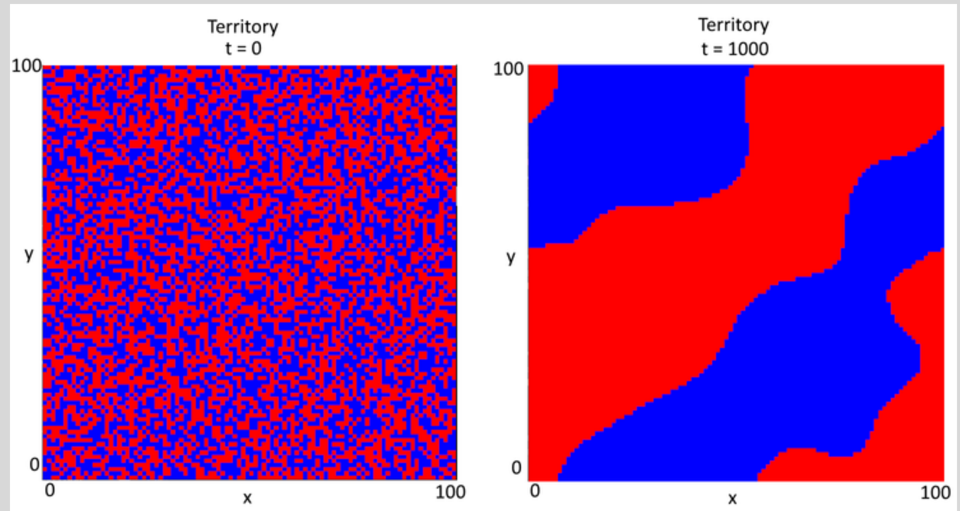

Fig. 6: Example of a stable segregated state by using a lower λ.