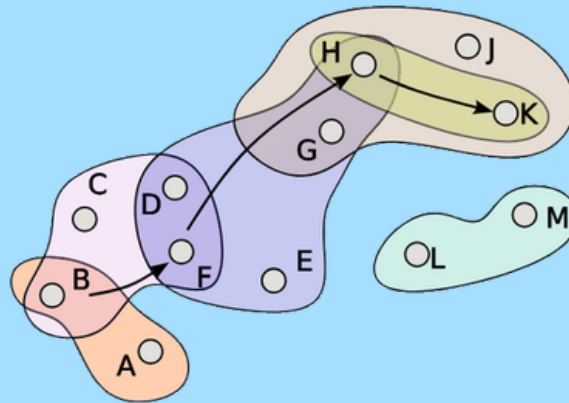


## 1. Background

- **Hypergraphs:** A generalization of graphs where hyperedges connect any number of nodes.
- **The Research Gap:** While prior studies focus on vulnerability of pairwise graphs, research on vital hyperlink properties for supporting routing is scarce.
- Our research focuses on networks that use shortest-path (SP) routing.

Figure 1: Hypergraph network and a shortest path from node B to node K. Adapted from Nortier et al. (2025), *Physical Review*.



## 2. Methodology

1. Define the 5 targeted strategies for link removal + random baseline.
2. Rank hyperlinks in each hypergraph, according to each strategy.
3. Gradually remove hyperlinks from hypergraphs, keeping track of the metric values at separate snapshots.
4. Compare the results to determine the best-performing strategy and the most critical hyperlink properties.

## 3. Strategies

- **Random:** Random order; used as a baseline.
- **Node-Degree Sum:** Sum of node degrees in the hyperlink.
- **Overlap Bridging:** Ties with the lowest Jaccard similarity to neighbors.
- **Participation Coefficient:** Links bridging the most communities.
- **Collective Influence:** Degree influence by neighbors of hyperlink nodes at distance  $\ell = 4$ .
- **Hyperlink Betweenness Centrality:** Sum of fractions of SPs passing through the hyperlink.

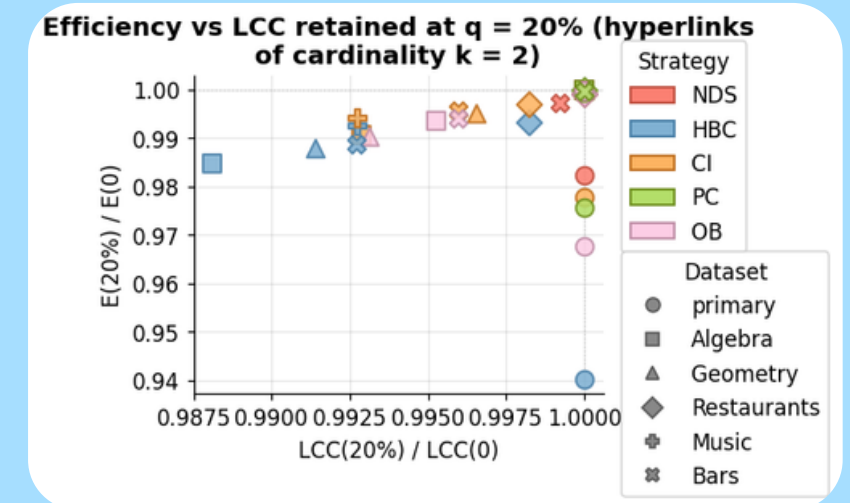
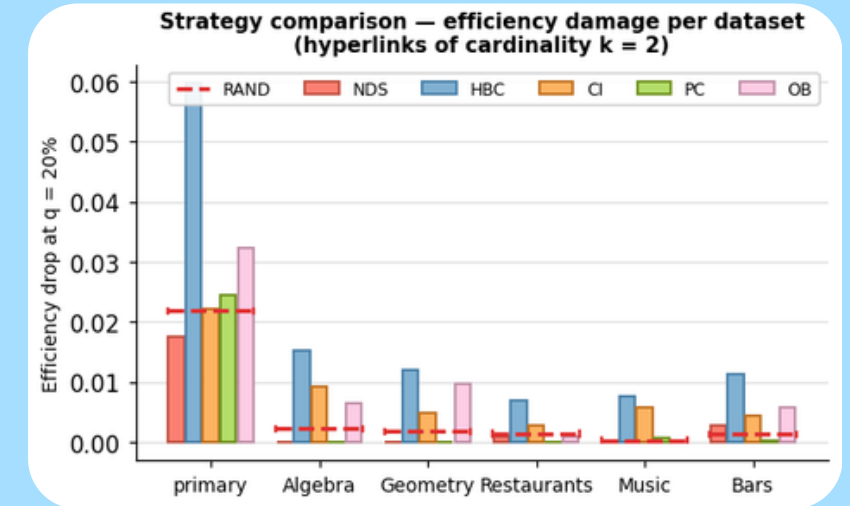
## 4. Metrics

- **Efficiency:** The average reverse SP distance between every pair of nodes. Distance is measured in terms of the number of traversed hyperlinks.
- **LCC fraction:** Fraction of the nodes in the hypergraph which are part of the largest connected component.

## 6. Conclusions

- HBC outperforms all other strategies.
- HBC manages to identify hyperlinks that are critical for maintaining connectivity, and thus disconnects regions of the hypergraph. Other strategies mostly rely on path stretching.
- NDS's failure shows that local structure provides very little information about SP transport in higher-order networks.
- Networks containing a substantial portion of large hyperlinks are more robust to transport attacks, due to the introduction of redundant connectivity.

## 5. Results (for $k = 2$ )



Spearman rank correlation between strategy pairs (rankings for hyperlinks of cardinality  $k = 2$ )

