Optimising Adaptive Resource Generation in Near-Term Quantum Networks

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Introduction

A quantum network uses entangled states to enable communication between quantum processors [1]. Entangled states (links) are a quantum network resource [2].

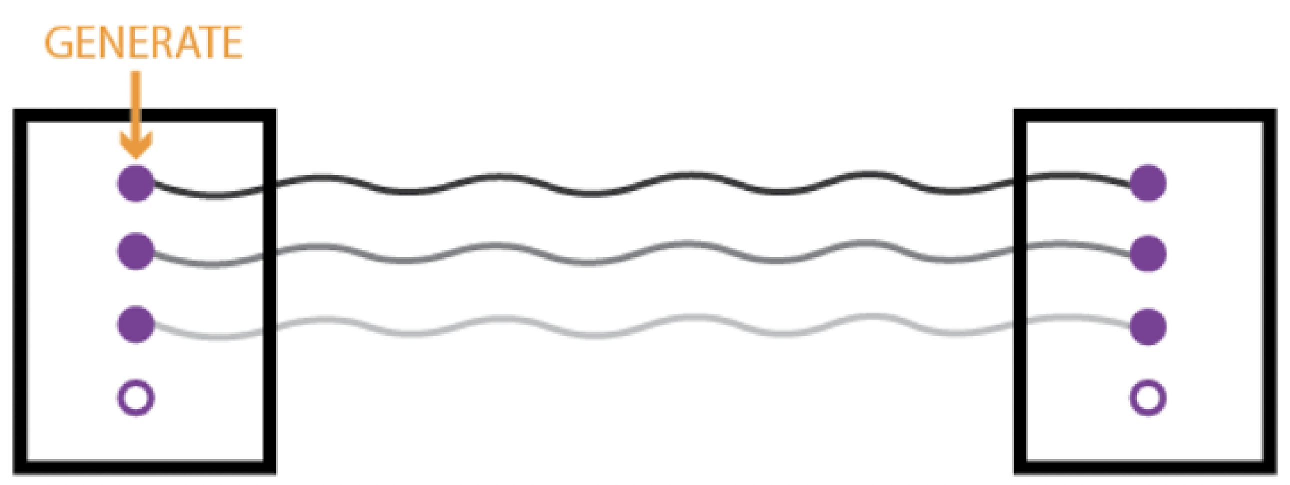


Figure 1. The fidelity of entanglement links in quantum memory decays exponentially. Courtesy of B. Davies.

Network Model

Quantum generation protocol (p, F)

- **Probability of success** (p): success rate of generation
- **Link fidelity** (F): how close is the link to the true Bell state

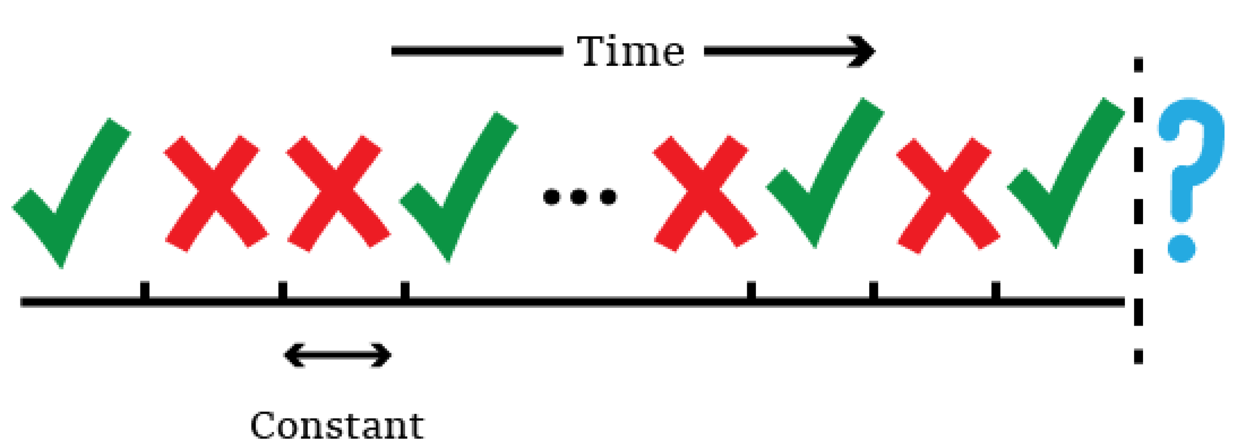


Figure 2. Process that attempts entanglement link generation at each time step. Courtesy of B. Davies.

Fidelity decay [2]

$$F \to \left(F - \frac{1}{4}\right) e^{-\Gamma} + \frac{1}{4}. \quad (1)$$

Research Questions

Questions

1. How to produce an optimal policy for generating n links by choosing from multiple protocols (p_i, F_i) at each time step?
2. What improvement does the adaptive optimal policy offer over a static process?

Methodology: Markov Decision Process

A **Markov decision process** is defined by a state space $\mathcal{S}$, an action space $\mathcal{A}$, transition probability p, and value function V [3].

Fidelity discretisation

Model fidelity dependent on the probability of success for the given protocol [4]:

$$(p_i, F_i) \mid F_i = 1 - \lambda p_i. \quad (2)$$

Fidelity discretised using the time t_i each generated link lives in memory (TTL) [2].

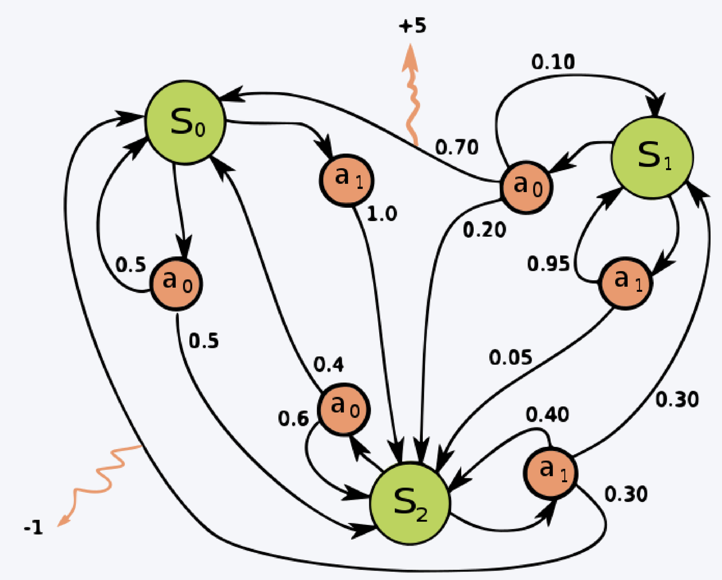


Figure 3. Markov decision process, courtesy of [5].

MDP definition

1. State space $\mathcal{S} = \{L_i = \{t_1, t_2, ..., t_m\} \mid m \in [0, n], t_i \in (0, \max t_i]\}$.
2. Action space $\mathcal{A} = \{p_i \mid p_i \in (0, 1)\}$.

Bellman equation

$$V_\pi(s) = -1 + \sum_{s' \in S} P(s'|s, \pi) \times V_\pi(s'), \forall s \in S. \quad (3)$$

Results: Policy Structure

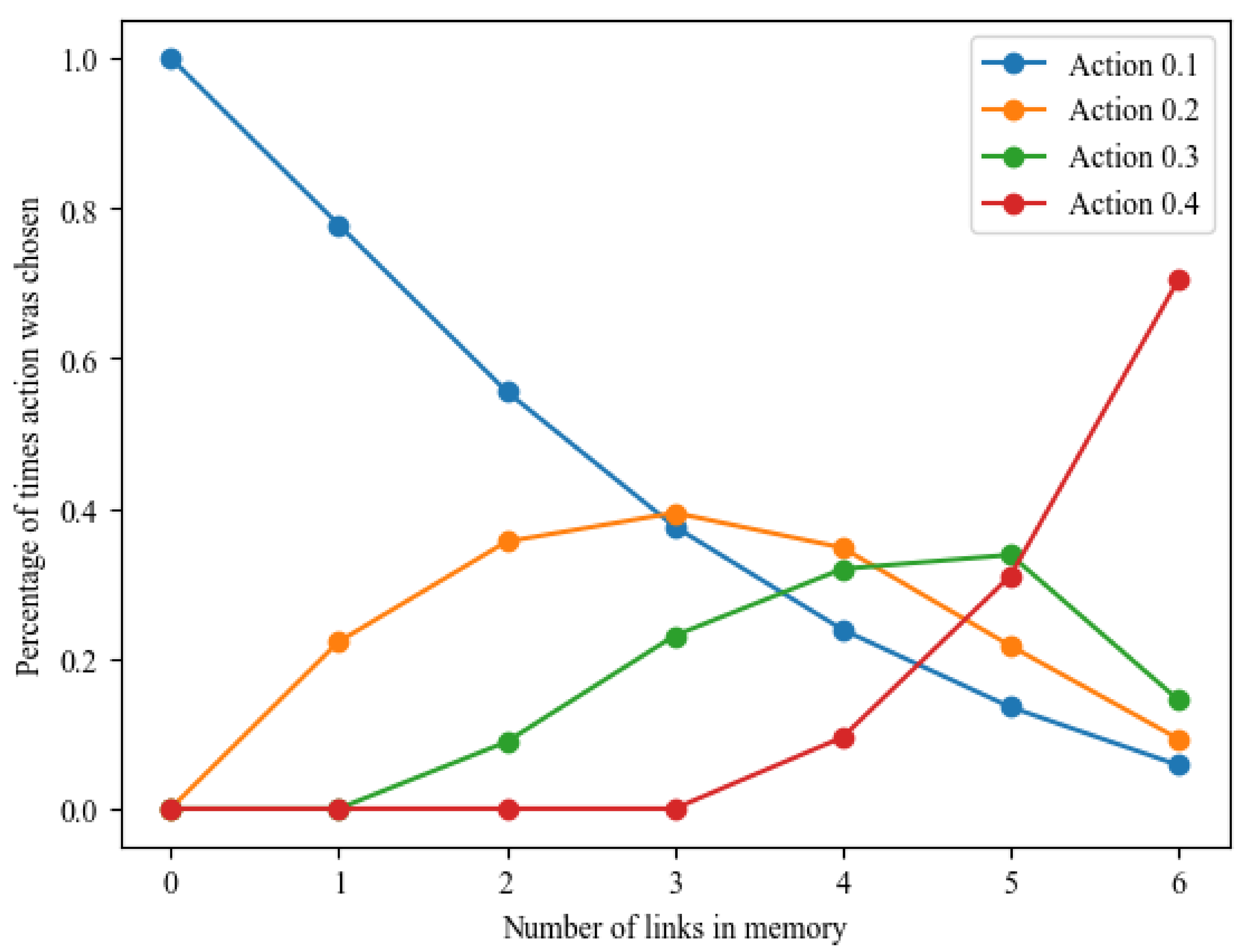
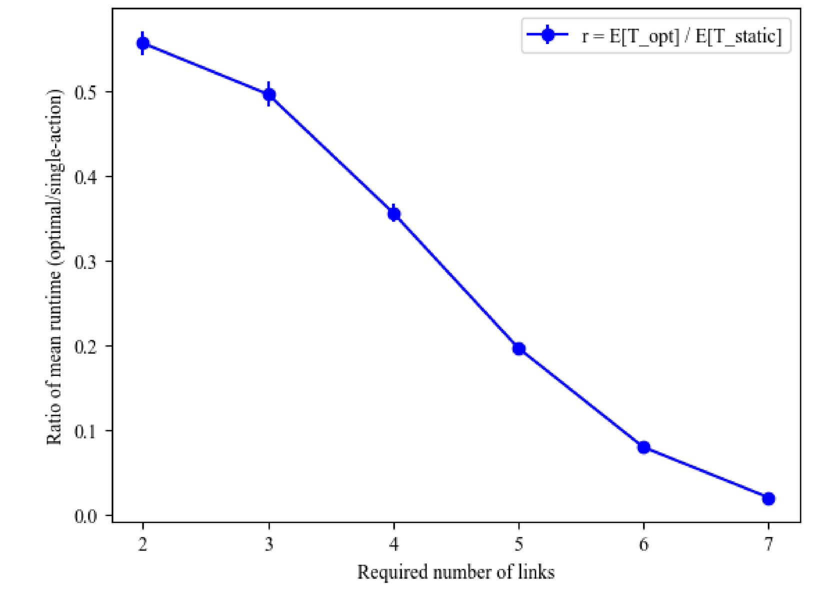
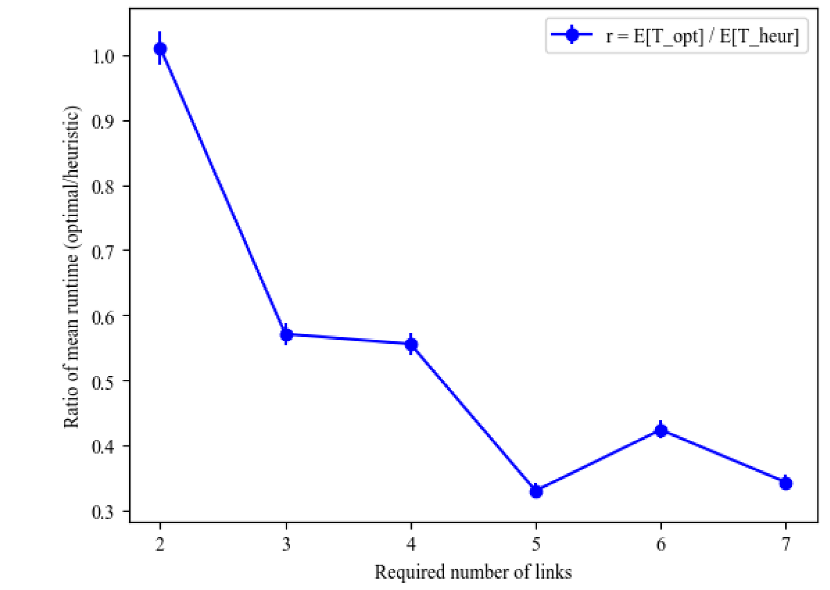


Figure 4. Actions picked by the optimal policy for all possible link counts in memory when $n = 7$. As the number of links in memory increases, the optimal algorithm picks protocols with higher probability p_i.

Results: Adaptive Policy Comparison



(a) Mean runtime of the optimal policy π vs. the best static policy (p = 0.2) for generating n links. Vertical lines show standard error.



(b) Ratio of mean runtime: optimal policy vs. policy using maximum allowable probability at each step. Vertical lines show standard error.

Conclusions

- The optimal policies gradually shift from actions with lower probability to actions with higher probability as the number of links in memory increases;
- As the number of required links increases, the gap between the optimal policy and our heuristic policy increases because the heuristic fails to capture the complexity of the process.

Limitations

- We consider applications that require only up to seven links due to the exponentially large cost of solving the MDP for larger search spaces;
- We have not considered any differences in the process time steps.

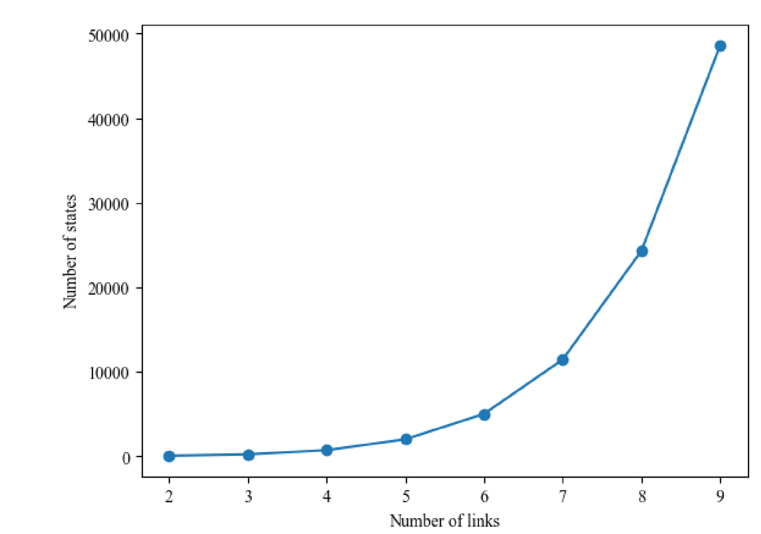


Figure 6. The number of MDP states grows with the number of required links in memory.

References

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