Learning Reduced Order Mappings of Navier-Stokes

An Investigation of Generalization on the Viscosity Parameter

A. Kiste[@*]

[@]a.kiste@student.tudelft.nl, [*]Faculty Electrical Engineering, Mathematics and Computer Science, TU Delft

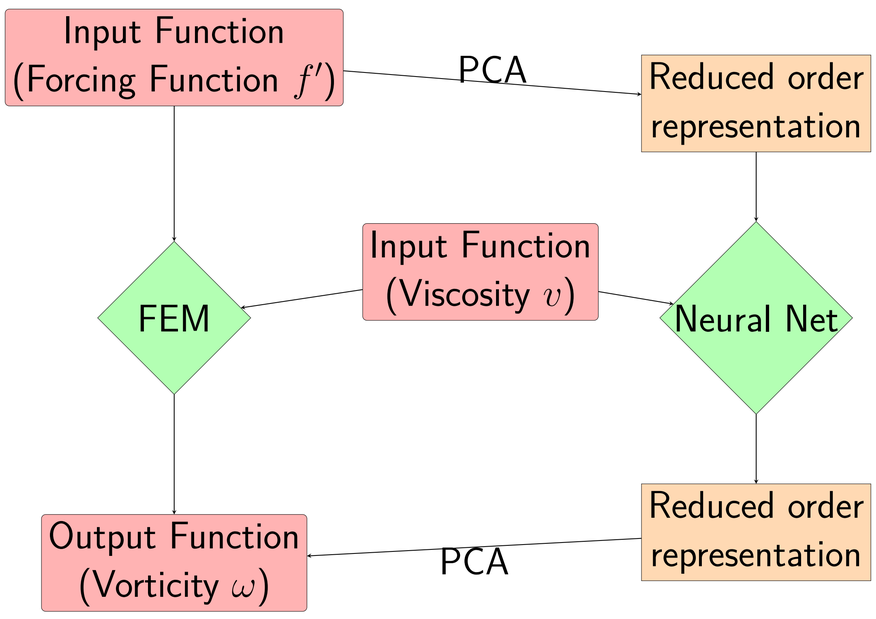
Background Information

- Partial Differential Equations (PDEs) describe the continuous changes of dynamic systems.
- Navier-Stokes is an expensive PDE to evaluate in engineering processes.
- High-dimensionality of input data makes neural networks expensive. Reduced-order could reduce costs.
- PCA-Net is a Neural Net architecture applying Principal Component Analysis on the input and output to work on a lower-dimensional data-set.

Research Question

To what extent can PCA-net generalize numerical solutions of Navier-Stokes equations on viscosity parameters?

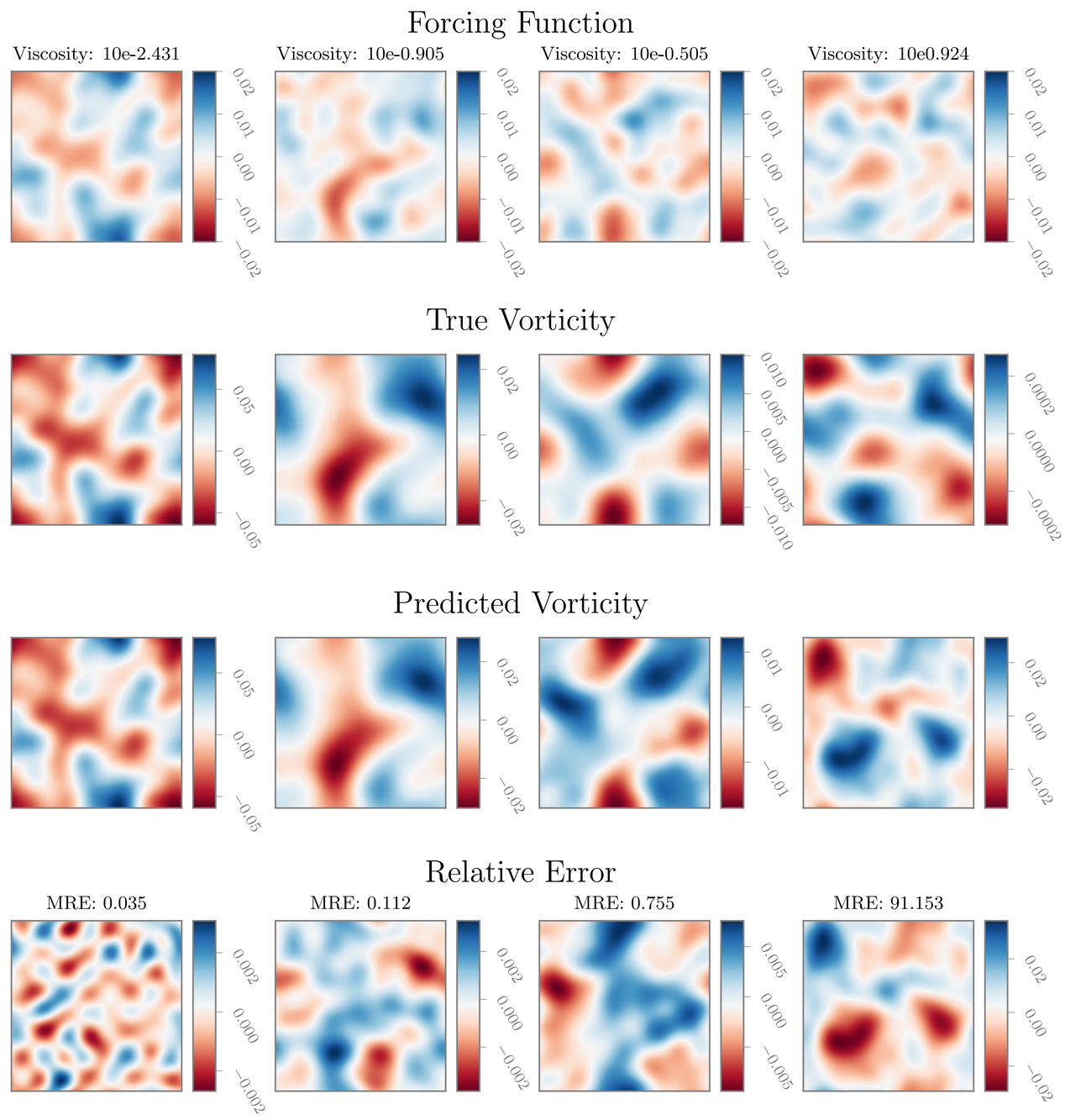
Methodology



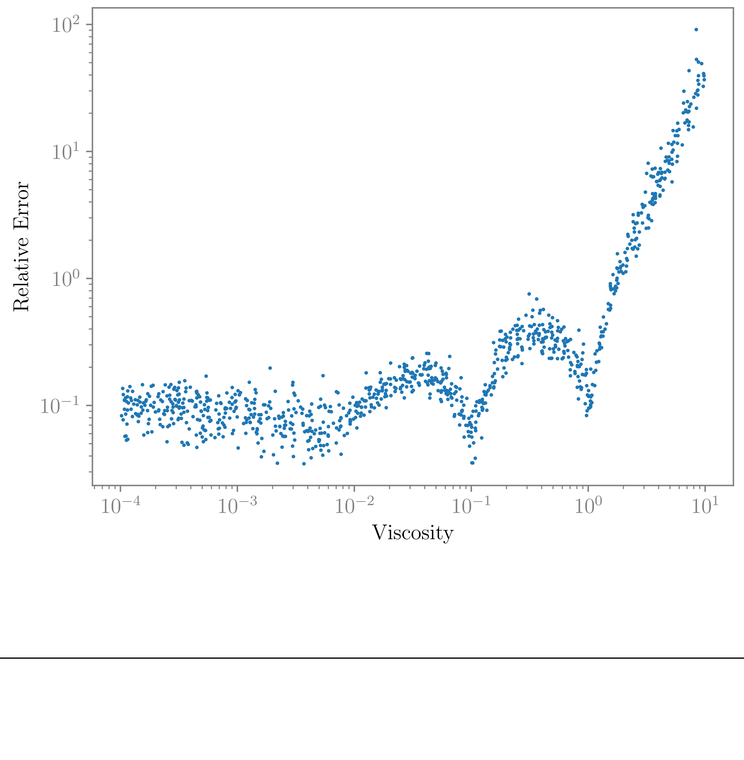
- Training done on four discrete viscosities with 5000 data points. 10^{-3}, 10^{-2}, 10^{-1}, and 10^{0}
- Testing done on continuous viscosities from 10^{-4} to 10^{1}

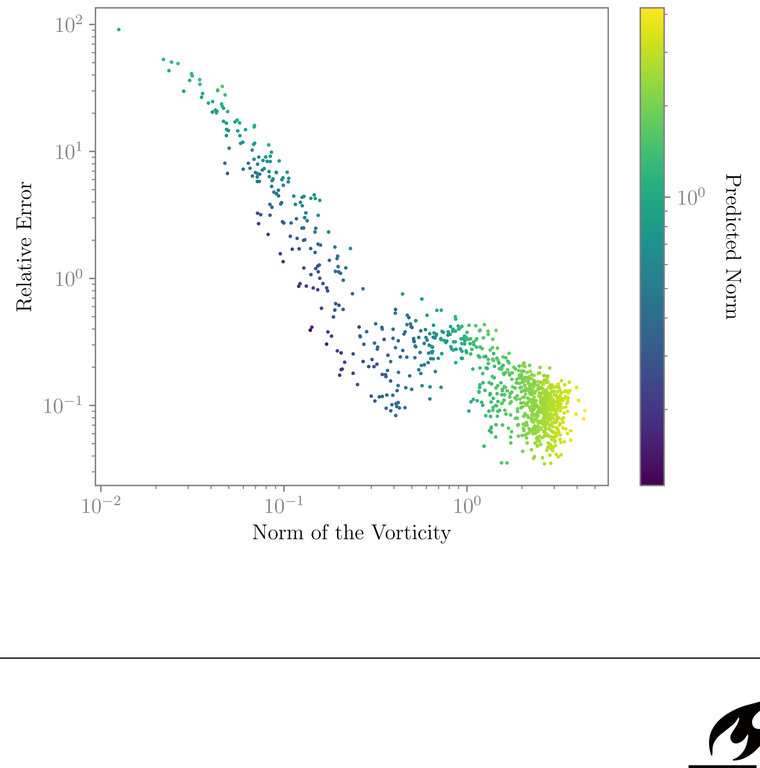
Results

- PCA-Net extrapolates information well on lower viscosities, with errors staying below $2 \cdot 10^{-1}$
- PCA-Net is unable to extrapolate at higher velocities, showing divergent error at viscosities above 10^{0}.
- PCA-net shows lower error when at training viscosities with higher errors when interpolating.
- Strong correlation between the norm of the vorticity and the relative error. Most data-points have a high norm.



Results





Conclusion & Future Work

- PCA-net can adequately generalize Navier-Stokes on viscosity, but not extrapolate higher viscosities.
- Future testing should include more training data on higher viscosities.
- Should be applied to more real-world applicable versions of Navier-Stokes. E.g flow around a 2d airfoil.
- Other neural operators should be run on the same test-set for side-by-side comparison.