Isomorphism is equality

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Research question:

How to formalise the results of the paper "isomorphism, is equality" into code to extend the UniMath library?

Prelimenaries

In order to create a formalisation we needed some preliminary theorems:

sigma equality :

x = y ≃ ∑ p : pr1 x = pr1 y, transportf A p (pr2 x) = pr2 y

Transport theorem:

∏ (p : P X), R X X ′ eq p = transportf P (weqtopaths eq) p

Results

Equality-pair-lemma

(C, , x, , p) = (D, , y, , q)
≃ (C, , x), , p = (D, , y), , q — associativity
≃ (C, , x) = (D, , y) — drop propositions
≃ ∑ eq : C = D, transportf (El a) eq x = y — sigma equality

Isomorphism is equality

∑ eq : C ≃ D, resp a C D eq x = y
≃ ∑ eq : C ≃ D, transportf (El a) (weqtopaths eq) x = y — transport theorem
≃ ∑ eq : C = D, transportf (El a) eq x = y — apply univalence
≃ (C, , x, , p) = (D, , y, , q) — apply equality-pair-lemma

Conclusions

- Defined a notion of isomorphism
- Proved the equality-pair-lemma
- Proved isomorphism is equality
- Defined a concrete universe
- Defined an alternative notion of isomorphism